FISHNET STATISTICS
FOR DESIGN OF QUASIBRITTLE
AND BIOMIMETIC MATERIALS AND
STRUCTURES FOR FAILURE
PROBABILITY <10^{-6}

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UNIVERSITY OF MIAMI, STRATEGIC RESEARCH INITIATIVES SEMINAR,
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Founder of structural safety. His work epitomized fusion of mechanics and probability.

After him: **50-year SCHISM:**
- advanced probability with simplistic mechanics
  or
- advanced mechanics with simplistic probability

Alfred M. Freudenthal
1906 – 1977
Overlooked: RELIABILITY-BASED DESIGN OF MATERIALS, NOT JUST STRUCTURES, AND FOCUSED ON THE TAIL

NEEDED: **TAIL-RISK DESIGN**

- $10^{-6}$ is the maximum tolerable $P_f$ for engineering structures

- Optimize not the mean material strength but the strength at the tail of $10^{-6}$ failure probability, $P_f$
Example: Huge tail difference between Gaussian (normal) and Weibull cumulative distribution functions (cdf)

Prob. of Failure, $P_f = 10^{-6}$

governs design

Assumed: same mean C.o.V.
In quasibrittle materials, for the same CoV, superior mean strength can lead to inferior strength at the $10^{-6}$ tail.

- The probability distribution must be known analytically!
- Controlling material architecture can profoundly alter the strength probability distribution.
Analytically Tractable Strength Models for Failure Probability (incl. tail)

EXISTING

1. a) Infinite weakest-link model
   —Weibull (1939) distribution; Fisher (1928)
   b) Finite weakest-link model (NU 2005)

2. a) Fiber bundle model (Daniels 1945)
   —Gaussian distribution
   b) Chain-of-bundles model
   (Harlow & Phoenix 1985)

NEW

3. Fishnet statistics (NU 2017)
Edmé Mariotte
1620 - 1684

Principal discoveries of the Church – 1653

Ronald Fisher
1928

Waloddi Weibull
1939

H. E. Daniels
1945
**Quasibrittle Materials**
— brittle constituents, but inhomogeneity size and the RVE are not $<<$ structure size $D$.

Concretes (archetypical, 1970s), tough ceramics, fiber composites, rocks, bones, sea ice, rigid foams, dental cements, dentine, cartilage, wood, consolidated snow, particleboard, paper, carton, cast iron, thin films, carbon nanotubes, fiber-reinforced concrete, cold asphalt concrete, mortars, masonry, stiff clay, silt, cemented sand, grouted soil, refractories, coal, oil and gas shales, nacre, biological shells, plus all brittle materials on micro- and nano-scales.

They all exhibit non-negligible material characteristic length.

At increasing size $D$, they all transition from ductile to brittle.
Today I focus on quasibrittle failures of Type 1 only.
I. Review of Recent Results on Tail Strength Probability of Quasibrittle Randomly Heterogenous Materials
The only way to determine $P_f$ is on atomistic scale:

**Frequency $\Rightarrow$ Probability**

Q = activation energy

$\Delta Q = \delta_a \left[ \frac{\partial \Pi(P, a)}{\partial a} \right]_P$

From Kramers’ rule of transition rate theory:

$$f_b \sim v_a \left( e^{-\left( Q_0 - \Delta Q / 2 \right) / kT} - e^{-\left( Q_0 + \Delta Q / 2 \right) / kT} \right) \approx 2v_a e^{-Q_0 / kT} \frac{V_a}{2E_a kT} \tau^2$$

How to upscale from nano to macro?
— scale transitions are governed by microcrack interactions in fracture process zone (FPZ)

- Power law tail, exponent \( n = 2 \) at nanoscale. In scale transitions to macro RVE, power law tail is indestructible, \( n \) is increased to 20—50. Parallel couplings increase \( n \), series couplings deepen tail.
Nano-Macro Transition of Tail of Strength cdf

- In parallel couplings (~ compatibility conditions across scales), the tail exponents are additive.

- In series couplings (~ localizations on subscale), they remain.

- Parallel coupling shortens the tail reach by order of magnitude.

- Series coupling extends the tail reach.

- Parallel coupling produces cdf with Gaussian core.

- **Power-law tail with zero threshold is indestructible!**

Distribution and Size Effect on Structure Scale

\[ P_f = 1 - \left(1 - P_{RVE}\right)^{N_{eq}} \]

**FINITE CHAIN**

\[ N_{eq} \to \infty \quad P_f = 1 - e^{-\left(\sigma / S_0\right)^m} \]

Infinite chain – Weibull distr. (not if quasibrittle)

\[ N_{eq} = \text{equivalent } N, \text{ as modified by the stress field or geometry} \]

\[ P_{gr} \]

Kink locations define the RVE size and \( P_{gr} \)

Calibration of $P_{gr}$ by the Size Effect

Note: Similar curves are predicted by deterministic nonlocal model.

Calibration Result: $P_{gr} \approx 0.001$

Note: Zero threshold!

Tail: $P_f \sim (\sigma - \sigma_u)^n$

Note: Zero threshold!

Prob. of Failure, $P_f$

$P_f = 10^{-6}$
governs design

Assumed: same mean C.o.V.

PLASTIC: Gaussian cdf

BRITTLE: Weibull cdf

As structure size increases, the tail risk point moves!

Tail-risk design must be size dependent
VALIDATION AND CALIBRATION: Optimal Fit of Weibull’s (1939) Tests (of Monumental Scope)

Portland cement mortars of 3 ages

Each point = average of ~100 tests

\[ \ln \sigma = \left( \ln \sigma_u - \ln(1-P_f) \right) \]

KINK — classical Weibull theory can’t explain (finite threshold is an incorrect remedy)

No test data exist for \( P_f < 0.005 \)

\[ \Rightarrow P_{gr} \approx 0.001 \]

RVE size ~ 0.6-1.0 cm;
Specimen vol. ~ 100-3000 cm³

Size Effect on Flexural Strength of Laminates
Reinterpretation of Jackson’s (NASA) Tests

Energetic-Statistical Size Effect Law:

Nominal Strength:
\[ \sigma_N = f_r^0 \left( \mathcal{G}^{n_d r/m} + \mathcal{G} \right)^{1/r}, \]

\[ \mathcal{G} = \frac{rD_b}{D + rsD_b} \]

Theory fits well!

- \( r = 0.8, \)
- \( m = 35, \)
- \( \omega = 0.135 \)

\( f_r^0, r, n_d, m, s, D_b = \text{constants}, \)

\( D = \text{char. size of structure}, \)

\( n_d = \text{no. of dimension for scaling} \)

\( m = \text{Weibull modulus, indepent of yarn layout} \)
Optimum Fit by Chain-of-RVEs, Zero Threshold (correct)

3-pt Bend Test, Porcelain

4-pt Bend Test on Dental Alumina-Glass Composite

4-pt Bend Test on Sintered Si₃N₄ with Y₂O₃/Al₂O₃ Additives

4-pt Bend Test on Sintered Si₃N₄ with CTR₂O₃/Al₂O₃ Additives

4-pt Bend Test on Sintered Si₃N₄

4-pt Bend Test on Dental Alumina-Glass Composite

Bažant, JL Le, MZ Bazant, PNAS 106 (2009), 11484
Optimum Fit by Weibull Theory with Finite Threshold —incorrect!

3-pt Bend Test on Porcelain (Weibull 1939)

\[ P_f = \frac{1}{1 - \sigma_u} \]

\[ \sigma_u = 13.4 \]

\[ \mu = 16.4 \]

S.F. = 1.22

ndata = 102

4-pt Bend Test on Dental Alumina-Glass Composite (Lohbauer et al., 2002)

\[ P_f = \frac{1}{1 - \sigma_u} \]

\[ \sigma_u = 230 \]

\[ \mu = 398 \]

S.F. = 1.73

ndata = 27

4-pt Bend Test on Sintered \( \alpha – \text{SiC} \) (Salem et al., 1996)

\[ P_f = \frac{1}{1 - \sigma_u} \]

\[ \sigma_u = 190 \]

\[ \mu = 361 \]

S.F. = 1.84

ndata = 107

4-pt Bend Test on Sintered \( \text{Si}_3\text{N}_4 \) with Y\(_2\text{O}_3/\text{Al}_2\text{O}_3 \) Additives (Santos et al., 2003)

\[ P_f = \frac{1}{1 - \sigma_u} \]

\[ \sigma_u = 586 \]

\[ \mu = 691 \]

S.F. = 1.18

ndata = 27

4-pt Bend Test on Sintered \( \text{Si}_3\text{N}_4 \) with CTR\(_2\text{O}_3/\text{Al}_2\text{O}_3 \) Additives (Santos et al., 2003)

\[ P_f = \frac{1}{1 - \sigma_u} \]

\[ \sigma_u = 577 \]

\[ \mu = 662 \]

S.F. = 1.15

ndata = 21

4-pt Bend Test on Sintered \( \text{Si}_3\text{N}_4 \) (Gross, 2003)

\[ P_f = \frac{1}{1 - \sigma_u} \]

\[ \sigma_u = 588 \]

\[ \mu = 733 \]

S.F. = 1.22

ndata = 27

\[ \ln \left( \frac{\sigma - \sigma_u}{\sigma_u} \right) \]

\[ \ln \left[ \frac{1}{1 - P_f} \right] \]

\[ S.F = 1.84 \]

\[ \mu = 361 \]

\[ S.F = 1.18 \]

\[ \mu = 662 \]

\[ S.F = 1.15 \]

Bažant, JL Le, MZ Bazant, PNAS 106 (2009), 11484
Wrong tail probability is experimentally provable only by size effect.
SIZE EFFECT ON MEAN STRENGTH

3-Parameter Weibull

Grafted 2-Parameter Gauss-Weibull

3D: $N_{eq}^{1/3}$
Malpasset Dam, failed 1959—size effect must have contributed

If designed today, the tolerable abutment displacement would have to be 51% smaller.
Fatigue of Nano-Scale Structure

Rate of failure of nano-structure at $\tau$:

$$f_b \sim v_a e^{-Q_0/kT} \int_{\alpha_0}^{\alpha_c} V(\alpha) d\alpha \frac{\tau(t)^2}{E_a kT}$$

Crack growth rate:

$$\frac{da}{dN} = \int_0^{t_c} \dot{a} dt \sim A e^{-Q_0/kT} f(R) \Delta \tau^2$$

Paris Law obtained, but $n=2$

Multiscale Transition of Mean Crack Growth Rate

Energy dissipation per cycle at nano-scale

\[ \Delta \Pi_a^* = e^{-Q_0/kT} f(R) \Delta K_i^2 \]

Macro-micro equality of energy dissipation per cycle:

\[ \Delta \Pi^* = \sum_{i=1}^{N} \Delta \Pi_i^* = N_a \Delta \Pi_a^* \]

\[ N_a = \text{no. of nano-cracks in the FPZ} = q_1 q_2 \ldots q_s \]

M. Salviato, K Kirane, & ZP Bažant (2014) JMPS, 440-454
Fits of Fatigue Lifetime Histograms of Ceramics

\[ \ln \{ \ln \left[ \frac{1}{1 - P_f} \right] \} \]

\[ \ln N_c \] (crit. number of cycles)

Yttria-stab. ZrO₂/feldspathic glass
1.6
1

Al₂O₃-ZrO₂/feldspathic glass
2
1

99% Al₂O₃
1.3
1

Histograms of Strength and Static Fatigue Lifetime

Calibration: Strength cdf

\[ \ln \{ \ln [1/(1-P_f)] \} \]

99.6% Al\(_2\)O\(_3\)
Fett and Munz, 1991

\[ P_{gr} \]

\( m \) = Weibull modulus for strength

\[ m_L = m/ (n+1) \]


Prediction: Lifetime cdf

\[ \ln \lambda \]

99.6% Al\(_2\)O\(_3\)
Fett and Munz, 1991

\[ \sigma = 0.78 \bar{\sigma}_N \]

Constant load

\( n \) = exponent of Charles-Evans law for subcrit. crack growth
Metals on micrometer scale: same pdf, same size effect in Poly-Si MEMS devices as concrete on meter scale

On-chip and slack-chain testers (Sandia, courtesy of B. Boyce)

Finite weakest link model:

\[ P_f = 1 - \left[ 1 - P_1(\sigma_N) \right]^n \]

\[ P_1(\sigma_N) = \int_0^\infty F_{f_t}(x\sigma_N)f_s(x)dx \]

Random tensile strength

Random stress field

II. New Results on Fishnet Statistics for Tail Strength Probability of Architectured Biomimetic (Nacreous) Materials
Nacre’s Nanostucture

Aragonite platelet

Organic matrix layer

Platelet surface

Organic matrix gap

Mineral bridge

Central region

Organic matrix sheet

Cross section

Aragonite platelet layer

F. Song et al. / Biomaterials 24 (2003) 3623–3631

5~10 μm

250~550 nm

21~31 nm

Lamella of Aragonite (95% Vol)
- a form of CaCO₃

Thin Bio-polymer Layer (5% Vol)
Idealization of Nacreous Nano–Architecture

Aragonite lamella

Shear Bond

Truss Link

Hinge

No Load Mechanism

Collapsed (for computations)

$P_f$ of leach ink is assumed to follow Gauss-Weibull graft

—resembles a fishnet pulled diagonally

https://en.wikipedia.org/wiki/Nacre

https://en.wikipedia.org/wiki/Nacre
New way to look at failure probability

— Prob. of survival:
  Union of disjoint sets $\rightarrow$ a sum):

\[
1 - P_f(\sigma) = P_{S_0}(\sigma) + P_{S_1}(\sigma) + P_{S_2}(\sigma) + \cdots \quad (P_{S_0} \gg P_{S_1} \gg P_{S_2})
\]

with 0 failed link \quad with 1 failed link \quad with 2 failed links

The first term represents the weakest-link statistics
Stress Redistribution in Fishnet Near Failed Zone (Laplace equation)

\[ \eta_i = \frac{\sigma_i}{\sigma_N} \]
Two-Term Fishnet Model

\[ N = m \times n \]

Fishnet

Two-Term Fishnet Model

\[ \nu_1 \text{ links with redistributed stress } \eta_a^{(1)} \sigma \]

\[ u \]

\[ \sigma \]

\[ N - \nu_1 - 1 \text{ links with original stress } \sigma \]

Stress redistribution: solved via Laplace equation

\[ P_{S_0}(\sigma) = [1 - P_1(\sigma)]^N \]

= joint prob. of survival (intersection of sets)

\[ P_{S_1}(\sigma) \simeq N P_1(\sigma) \cdot [1 - P_1(\sigma)]^{N-\nu_1-1} \cdot [1 - P_1(\eta_a^{(1)} \sigma)]^{\nu_1} \]

Any one of N links must already have failed

survival of remaining N-1 links

joint probability

equivalent redistributed stress


In brief: PNAS 2017, 12900
Reinterpretation of Two-Term Fishnet Model

\[ 1 - P_f(\sigma) = P_{S_0}(\sigma) + P_{S_1}(\sigma) \]

\[ 1 - P_f(\sigma) = [1 - P_1(\sigma)]^N \left\{ 1 + \frac{NP_1(\sigma) \cdot P_\Delta}{\underbrace{1 - P_1(\sigma)}} \right\} \]

Weakest Link Model

\[ P_\Delta = \frac{1}{1 - P_1(\sigma)} \cdot \left[ \frac{1 - P_1(\eta^{(1)}_\alpha(\sigma))}{1 - P_1(\sigma)} \right]^{\nu_1} \]

Conditional Prob. of survival for the \( \nu_1 \) links

Under redistributed stress

\( P_{S_1}(\sigma) \) is important only for lowermost tail of \( P_f \)!

Doubled slope—advantageous

\[ P_f \text{ in Weibull scale:} \]

Lower Left Tail:

\[ \ln[-\ln[1 - P_f(\sigma)]] = 2m \ln \sigma + \ln \left[ \frac{n(n+1)}{2} \right] + 2 \ln c \]

\[ Y = A \cdot X + B \]

Upper Right Tail:

\[ \ln[-\ln[1 - P_f(\sigma)]] = m \ln \sigma + \ln n \]
Three-Term Fishnet Model

\[ 1 - P_f(\sigma) = P_{S_0}(\sigma) + P_{S_1}(\sigma) + P_{S_2}(\sigma) \]

Same as before

2 cases: \[ P_{S_2} = P_{S_{21}} + P_{S_{22}} \]

The second failure, if adjacent to the first one, if not

failure of two links — joint prob. with survival of remaining links

\[ P_{S_{21}} = N\nu_1 \left\{ P_1(\sigma)P_1(\eta_1^{(1)}\sigma) - \frac{1}{2}[P_1(\sigma)]^2 \right\} \cdot [1 - P_1(\sigma)]^{N-\nu_2-2} \cdot [1 - P_1(\eta_2^{(2)}\sigma)]^\nu_2 \]

\[ P_{S_{22}} = \frac{1}{2}N(N - \nu_1 - 1)[P_1(\sigma)]^2 \cdot [1 - P_1(\sigma)]^{N-2\nu_1-2} \cdot [1 - P_1(\eta_2^{(2)}\sigma)]^{2\nu_1} \]

\[ P_{S_{22}} \gg P_{S_{12}} : \text{second damage occurring far away from the first} \]

No longer negligible when the tail of \( P_1 \) is thick or CoV high

W Luo, ZP Bažant, 2017 Fishnet Statistics for Strength Scaling of Nacre-Like Imbricated Laminar Materials, PNAS & JMPS
Verification by 1 million Monte Carlo simulations
(about $10^4$ simulations per data point near center)

4.5–fold increase!
— enormous enhancement of safety
Verification by 1 Million Monte Carlo FEM Simulations

When $P_1(\sigma)$ has a thin power law tail:

Typical Load – Displacement Curve

Damage Evolution

$k$: Number of failed links

Single Crack, no scattered damages.
Verification by 1 Million Monte Carlo FEM Simulations

- **Weakest Link Model**
- **2-Term Fishnet**

Sample Size=$10^6$

Tail Enlarged

Lower tail of $P_f$ is **thin**

Extremely weak links are **rare**

Most likely, failure localizes right after the **first** damage

Analytical prediction matched

$P_f = 0.5$

Extremely weak links are more common. Most likely, crack localizes after a few scattered damages.

Verification by 1 Million Monte Carlo Simulations

16 x 32 fishnets

Three-term fishnet
Two-term fishnet
Weakest link

Lower tail of $P_1$ is thick

Extremely weak links are more common

Most likely, crack localizes after a few scattered damages
Enormous effect of fishnet architecture on safety:

<table>
<thead>
<tr>
<th></th>
<th>$\ln \sigma$</th>
<th>$\sigma$ /MPa</th>
<th>$P_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weakest-Link Model</td>
<td>1.8</td>
<td>6.05</td>
<td>$29.5 \times 10^{-6}$</td>
</tr>
<tr>
<td><strong>Two-Term</strong> Fishnet</td>
<td>1.8</td>
<td>6.05</td>
<td>$1.19 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

— 25-fold decrease of failure probability $P_f$ at cdf tail! (and more for more terms and higher scatter)

Chain $1 \times N \rightarrow k/n \rightarrow 1/N$

Fishnet $k/n \rightarrow N/1$

Bundle $N \times 1$

Upper Bound $\rightarrow$ Increasing Reliability $\rightarrow$ Lower Bound

$N = kn$

**Shape effect - enormous on tail**

Weibull Scale

$P_f = 0.5$

$P_f = 10^{-6}$

Weibull-to-Gaussian Transition of cdf upon Changing Aspect Ratio of Fishnet

Upper Bound ➔ Increasing Reliability ➔ Lower Bound

Effect of CoV of Link Strength Scatter at Tail

Low scatter – one crack – weakest link applies

High scatter – many cracks – doesn’t → safer at tail!
Fishnet Calibration: via Statistical Size Effect

— similar to Type 1, but a set of intermediate asymptotes

\[ \log(\text{strength}) \text{ versus } \log(\text{size } D) \]
Statistical Size Effect
(shape $k \times n$ kept constant)

— it is an envelope of a series of asymptotes, each obtained from one term in the probability sum
III. Latest Results at NU on Fishnet Statistics
Failure Probability of Softening Fishnet

\[ P_f(x) = \mathbb{P}(\sigma_{max} \leq x) = \sum_{k=0}^{N} \mathbb{P}(N_c = k) \mathbb{P}(\sigma_{max} \leq x \mid N_c = k) \]

\( N_c \): Num. of Damaged Links at Peak Load

Geometric Poisson Distribution (Pólya-Aeppli 1924)
(for random cluster of damages)

Order Statistics
\( (k^{th} \text{ smallest minimum of link strength}) \)

\[ W_k(x) = \mathbb{P}[S(k) \leq x] \]

\[ W_k(x) = 1 - [1 - W_0(x)] \sum_{s=0}^{k} \left\{ -\ln[1 - W_0(x)] \right\}^s / s! \]

\[ p_k = \mathbb{P}(N_c = k) = \begin{cases} \sum_{s=1}^{k} \frac{e^{-\lambda} \lambda^s (s-1)! \theta^s (1 - \theta)^{k-s}}{s!}, & k = 1, 2, 3, \ldots \\ e^{-\lambda}, & k = 0 \end{cases} \]
Let $N_c = \text{number of damaged links at max. load}$

$$P_f(x) = \mathbb{P}(\sigma_{max} \leq x) = \sum_{k=0}^{N} \mathbb{P}(N_c = k) \mathbb{P}(\sigma_{max} \leq x \mid N_c = k)$$

Distribution of $k^{th}$ smallest minimum, $s_{(k)}$, of link strength:

**Order Statistics:** $N_c$ follows $W_k(x) = \mathbb{P}[s_{(k)} \leq x] = \text{geometric Poisson distribution (Pólya-Aeppli), for random cluster of damages: } N_c$ follows
Order Statistics (k-th smallest minimum):

\[ W_k(x) = \mathbb{P}[S_{(k)} \leq x] \]

\[ W_k(x) = 1 - [1 - W_0(x)] \sum_{s=0}^{k} \frac{(-\ln[1 - W_0(x)])^s}{s!} = \mathbb{P}[S_{(k)} \leq x] \]

Wen Luo & ZP Bazant (2018), JMPS 281—295
Localization Indicator \( \gamma_k = \frac{s(k)}{\sigma_k} \)

\[ K_t = -0.1 K_0 \]
\[ J = 20 \]

Divergence point indicates max. load

\( s(k) = \) loading stress corresponding to \( k \)
Further Strength Gain at 10^{-6} Probability Tail Due to Gradual Postpeak Softening

\[ P_f(x) = \sum_{k=k_0}^{N} p_k W_{k'} \left( \gamma_{k'} \frac{x}{k'} \right), \quad k' = k - \delta k \]

---constitutive law is crucial to failure prob.!

Lower softening slope \( K_t \) – beneficial

50% Tail Strength Increase

Wen Luo & ZP Bazant (2018), JMPS 281—295
Fit of Distribution of Number $N_C$ of **Damaged Links** at Peak Load by cdf of Geometric-Poisson Distribution – Very Close!

Note: It also proves that stress redistribution from each of many drops can be neglected

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Wen Luo & ZP Bazant (2018), JMPS 281—295
Fit of Number of Damaged Links $N_c$ at Peak Load by pdf of Geometric-Poisson Distribution, for Various Softening Slopes
Size Effect = Joint Effect of Horizontal and Vertical Scaling
Simulated Sample Size = $10^4$

**Longitudinal Scaling**

**Weakest-link rule** – the histogram shifts up by $\ln\left(\frac{s_2}{s_1}\right)$ if the length is increased from $s_1$ to $s_2$;

**Transverse Scaling**

Histograms rotate about a point, $Q$, at a constant rate, equally for each doubling of width → **Slope increases**.
Inferring Fishnet Strength Distribution from Size Effect

Strength Distribution: \( Y - y_0 = m_0\left[1 + c \ln\left(\frac{r}{r_0}\right)\right](X - x_0) + \ln\left(\frac{s}{s_0}\right) \)

Median Size Effect:

\[
\ln \sigma_{0.5} = \frac{\ln \ln 2 - y_0 - \ln D}{m_0(1 + c \ln D)} + x_0
\]

Parameters:
- \( c = 0.27 \)
- \( m_0 = 32 \)
- \( x_0 = 2.03 \)
- \( y_0 = -1.3 \)

Reverse Size Effect!
Rate Dependence of Strength Statistics (J.-L. Le et al.)

Rate dependent weakest-link model:

\[ P_f(\sigma_N, \dot{\varepsilon}) = 1 - [1 - P_1(\sigma_N, \dot{\varepsilon})][1 - P_1(\sigma_N, \dot{\varepsilon})]^{n(\dot{\varepsilon})-1} \]

Stochastic DEM simulation of dynamic fracture of aluminum nitride:

Found: Weibull modulus \( m \) increases with strain rate.

Fishnet for Rate-Dependent Strength Statistics of Aluminum Nitride (J.-L. Le et al., JAM 2018)

As strain rate increases, more grain boundaries get damaged at peak load \( \rightarrow \) more fishnet terms dominate \( \rightarrow \) increase of \( m \)

\[
F_s(\sigma_N) = 1 - [P_{s,0}(\sigma_N) + P_{s,1}(\sigma_N) + \cdots + P_{s,N}(\sigma_N)]
\]

\[
P_{s,k}(\sigma_N) = \text{prob. that number } k \text{ of grain boundaries experience damage at stress}
\]

Stochastic DEM

Higher rate \( \rightarrow \) more diffused cracking at the peak load

As strain rate increases, more grain boundaries get damaged at peak load \( \rightarrow \) more fishnet terms dominate \( \rightarrow \) increase of \( m \)
To sum up:

- For quasibritile materials, we need **TAIL-RISK DESIGN** not Mean-Based Design.
- The safety factor is size dependent, and the reliability indices (Cornell, Hasofer-Lind) need to be modified.
- No test data exist for $P_f < 0.005$. Are concrete, composites partially “fishnet”?
For quasibrittle materials, and esp. architectured and biomimetic ones:

| Error | in safety factors | >> | Error | in deterministic analysis |

because the devil is in the tail
For quasibrittle materials, **TAIL-RISK DESIGN** not Mean-Based Design

**Thanks for Listening!**

Google “Bazant”, download 455, 484, 485, 486, 488, 489, 491, 508, 509, 514, 530, 532, **542, 554, 560, 557, 562, 584, 590** (all .pdf) and summary papers: **464.pdf, 500-501.pdf; 583.pdf; 590.pdf**