Supply Contracts in Manufacturer-Retailer Interactions with Manufacturer-Quality and Retailer Effort-Induced Demand

Haresh Gurnani,1 Murat Erkoc2

1 Department of Management, University of Miami, Coral Gables, Florida 33124
2 Department of Industrial Engineering, University of Miami, Coral Gables, Florida 33124

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Abstract: We consider a decentralized distribution channel where demand depends on the manufacturer-chosen quality of the product and the selling effort chosen by the retailer. The cost of selling effort is private information for the retailer. We consider three different types of supply contracts in this article: price-only contract where the manufacturer sets a wholesale price; fixed-fee contract where manufacturer sells at marginal cost but charges a fixed (transfer) fee; and, general franchise contract where manufacturer sets a wholesale price and charges a fixed fee as well. The fixed-fee and general franchise contracts are referred to as two-part tariff contracts. For each contract type, we study different contract forms including individual, menu, and pooling contracts. In the analysis of the different types and forms of contracts, we show that the price only contract is dominated by the general franchise menu contract. However, the manufacturer may prefer to offer the fixed-fee individual contract as compared to the general franchise contract when the retailer’s reservation utility and degree of information asymmetry in costs are high. © 2008 Wiley Periodicals, Inc.


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1. INTRODUCTION AND RELATED LITERATURE

In decentralized distribution channels, the manufacturer relies on marketing intermediaries (such as retailers) to take the product to the end-user. However, each firm makes decisions to maximize their individual profits and hence, the presence of disparate incentives for the firms requires careful consideration in the design of supply contracts. Two basic issues commonly identified in supply contract design are the double marginalization effect [28] and information asymmetry. In this article, we assume that the manufacturer designs the contract but both the manufacturer and retailer engage in activities that influence final demand.

For instance, in the electricity markets, power generating firms can invest in different dimensions of power quality such as environmentally-friendly green power or premium power for sensitive computing, as opposed to lower quality interruptible power for flexible producers [27]. In the fast food retail business, final demand is not only affected by the retail price and the value added by the retailer, it also depends on the investments made by the franchisor in its brand name [21]. In the automotive industry, Japanese firms made dramatic gains in market share in the 1980s as compared to the US firms by investing in quality-improvement efforts [12].

Similarly, the retailer has the opportunity to influence the final demand by choosing the appropriate selling/promotional efforts. Examples of such activities include breaking bulk, providing shelf spaces, promotional displays, advertising, and other demand enhancing activities [10]. In the automotive industry, final demand depends not only on the quality/brand reputation of the product, but also on the dealer’s selling efforts including after-sales service support. The cost of these promotional efforts is private information and is directly incurred by the retailer only. In many situations, the manufacturer may not know the retailer’s effectiveness (or willingness) to exert effort to influence demand.

Correspondence to: H. Gurnani (haresh@miami.edu)
For instance, in electronic commerce settings, a manufacturer selling to a first-time buyer (retailer) is unlikely to know the buyer’s ability to use effort to develop the market. In a retail setting, the manufacturer may not know the incentives for the retailer to set the appropriate shelf space for his product in the presence of different profit margins from competing products. Moreover, since the manufacturer may not be able to reliably verify the retailer’s effort, we assume that retailer effort is noncontractable in the model. If the retailer decides to accept the contract terms offered by the manufacturer, she has the flexibility to jointly choose the effort level and the retail price in order to maximize her own profits.

1.1. Types of Contracts

We compare three different types of supply contracts in this paper.

1. At one extreme is the wholesale price contract with an arm’s length relation between the manufacturer and the retailer; the manufacturer invests in product quality and the retailer buys the product at the wholesale price and sells it to the consumers with no obligation to the manufacturer. The retailer may accept the contract terms in which case she would choose her selling effort along with the retail price. Wholesale price contracts are easy to implement and have been extensively used in many industries including for electricity, electronics, food items, pharmaceuticals, etc. Lariviere and Porteus [22] note that in many supply chains, transactions are “governed by simple contracts defined only by a per unit wholesale price.”

2. In the second contract type, referred to as the fixed-fee contract, the manufacturer invests in product quality, sells the product at cost to the retailer but charges a fixed-fee (transfer fee) to the retailer. Note that in the fixed-fee contract, since the wholesale price is set equal to the marginal cost, the manufacturer derives all of his profits by careful selection of the transfer fee. Essentially, the transfer fee serves two main purposes: It is used as a mechanism to divide the total channel profits; further, the retailer seeking a high-quality product may be willing to pay a higher transfer fee in order to reimburse the manufacturer for the cost of quality-improving activities. Again, on acceptance, the retailer chooses the selling effort and the retail price.

3. Finally, we consider the general franchise contract where the manufacturer can select the wholesale price along with the transfer fee. Franchise contracts have had remarkable success and it is estimated that more than 40% of retail sales in the United States pass through some franchised operations [7, 19, 20]. While the wholesale price contract is a single-tariff price-only contract, the fixed-fee and franchise contracts are two-part tariff contracts with the fixed-fee contract as a special case of the general franchise contract.

1.2. Forms of Contracts

While the retailer’s cost of effort is not known to the manufacturer, the manufacturer has some priors on the retailer being efficient with low costs (l-type) or with high costs (h-type). For instance, a retailer with a strong market presence may be more cost-effective in influencing demand as compared to a retailer with a weaker presence. We consider the following three forms of contracts:

1. If the manufacturer has perfect information about the retailer type, he can offer the optimal contract derived in the perfect information case. In the imperfect information case, the manufacturer may still use the same contract form derived for the perfect information case by assuming that the retailer is of h-type or of l-type. Essentially, here the manufacturer does not attempt to identify the retailer type but offers a contract form which would be optimal if the retailer type is the one as originally assumed. We refer to such contract form as an individual contract.1

2. We also consider a second form referred to as menu contract with self-selection. The menu contract is a separating equilibrium contract where each retailer selects the contract expressions designed strictly for her type. Here, the manufacturer offers a menu of contracts (with self selection) in order to distinguish between the retailer type. However, the contract structure has some loss in efficiency because of the prospect of information rent collected by the l-type retailer.

3. Finally, we consider a pooling contract where, similar to the individual contract case, the manufacturer offers a single contract, but now the contract must satisfy the participation constraint for both types of retailers. The individual contract considered

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1 The motivation for using the Individual Contract lies in the fact that the structure of the contract is efficient in the perfect information case and we check whether it may be the preferred one even in the imperfect information case. Consider the case when the manufacturer offers the low-cost Individual contract assuming that the retailer is of the low-cost type. If the retailer is indeed of the low-cost type, the contract form is optimal for the manufacturer. On the other hand, if the retailer is of the high-cost type, she may still choose to accept the terms offered in the contract. The expected profit for the manufacturer would then depend on the probability of the retailer types. Similarly, if the manufacturer offers the high-cost Individual contract, the low-cost retailer may also be willing to accept the contract terms.
above is different from the pooling contract as it is designed for one retailer type and does not consider the individual rationality constraint of the other.

In the analysis of the various types/forms of contracts, we first compare within each contract type for the different contract forms. Subsequently, we compare across the different contract types.

### 1.3. Research Objectives and Managerial Implications

Since the manufacturer is assumed to be the leader in this article (the one who designs the terms of the contract), the problem is to choose the contract that maximizes his expected profits and ensures that the retailer participates by doing at least as well as her reservation utility. The limitation of wholesale price contracts is the double marginalization effect leading to lower profits for both firms. For the case of perfect information, we know that the fixed-fee contract is channel optimal and would always dominate the wholesale price contract [24]. However, when the retailer’s cost of effort information is private knowledge, and both the manufacturer and the retailer make decisions not only on prices but on other demand-affecting variables as well, would the wholesale price contract ever be used? In addition, would the fixed-fee contract (which is channel optimal under perfect information) be preferred to the general franchise menu contract (with both wholesale price and transfer fees) in the imperfect information case?

In the comparison of the different contracts, we are able to generate managerial insights related to the type and form for the contract selection problem for the manufacturer. The following results are established in the paper:

- We show that the pooling contract is always dominated by either the individual or the menu contract for all types of contracts;
- If the manufacturer is constrained to offer contracts with only wholesale prices, then he always prefers to offer a menu of contracts over the individual contract;
- If the manufacturer is constrained to offer the fixed-fee contract, then it may prefer to offer the individual contract over the menu of contracts under certain conditions;
- It is never optimal for the manufacturer to exclude transfer fees in the franchise menu contract, that is, the wholesale price menu contract is dominated by the franchise menu contract. It is important to note that even though the wholesale price menu contract is dominated, we need the results derived above (that wholesale price individual contracts will never be preferred to the wholesale menu contract) to eliminate the wholesale price type contract.
- Given a choice between the general franchise menu contract and the fixed-fee contract, under certain conditions, the manufacturer may prefer the fixed-fee individual contract. Here, the manufacturer would exclude the \( h \)-type retailer and offer fixed-fee individual contract that is accepted by the \( l \)-type retailer only.

We show that when the manufacturer’s prior on the retailer being of the \( l \)-type exceeds a certain threshold, it may be preferable to offer the fixed-fee individual contract as compared to the franchise menu contract. Intuitively, the individual contract is efficient for the case of perfect information and as such, the manufacturer may prefer it to the franchise menu contract (which has some loss in efficiency in order to make it self-selecting for both retailer types) when the likelihood of the retailer being of the \( l \)-type is high. Further, we also note that the preference for the fixed-fee individual contract increases when the reservation utility of the retailer and the degree of asymmetry in costs is high. As such, the manufacturer becomes more willing to exclude the high-cost retailer by offering the fixed-fee contract which is only accepted by the \( l \)-type retailer.

To the best of our knowledge, there is no similar study of the contract selection problem for a manufacturer using different types (wholesale, fixed-fee, and franchise) and forms (individual, pooling, and menu) of contracts when both players can use price and nonprice factors to influence demand.

### 1.4. Related Literature

The problem of contract design in decentralized distribution channels (supply chains) is of interest to researchers in the operations management, marketing, and economics communities. We briefly review some of the key papers in different streams of research in this area.

Cachon [3] provides an excellent review of different aspects of the operations management literature as it pertains to supply chain contracts. In a article with information asymmetry, Corbett et al. [6] investigate various two-part tariff contracts under a one-supplier, one-buyer setting where the buyer’s cost is private information. Their study focuses on supplier-initiated contracts and evaluates the value of information for the supplier. In their article, the supplier does not know the buyer’s internal variable cost; however, they do not consider supplier or buyer investment decisions to influence the demand through higher quality or selling efforts, respectively. In line with their article, we observe that as the information gap about retailer type increases, the flexibility provided by using transfer fees is reduced. However, we also note that the manufacturer may prefer the fixed-fee contract over the general franchise menu contract (two part tariff contract) under certain conditions. In a paper with retailer selling...
effort, Krishnan et al. [18] consider the effect of retailer promotions on demand in a stochastic model. They discuss that a buy-back contract alone cannot coordinate the channel as buy-backs reduce the incentive for retailer’s promotional efforts. As such, they suggest using buy-backs coupled with promotional cost-sharing agreements when effort cost is observed in order to coordinate the channel. Other coordinating mechanisms for the case of observable (but not verifiable) demand, and for the case of verifiable demand are also discussed. Other research in this stream examines the role of a variety of mechanisms to improve supply chain efficiency including buy-back agreements [26], quantity commitment contracts [1], quantity discount contracts [5], information sharing [13], and revenue sharing [2, 4, 14].

Researchers in marketing and economics have modeled the role of wholesale prices on the supply contract problem. We refer to the seminal work by McGuire and Staelin [23] and Jeuland and Shugan [17] for additional information. Lal [21] examines franchising arrangements in improving channel coordination by focusing on two elements in the contract – royalty structure and monitoring technology. The results indicate that monitoring is necessary if there is franchise competition to prevent the free riding problem. When the franchisor also invests in the brand name that influences the final demand, the optimal arrangement requires the use of royalties. Gal-Or [11] studies the role of monitoring in maintaining quality standards in franchise chains. The findings of the work indicate that the franchisor has greater incentive to monitor the service efforts of franchisees that serve relatively small markets, and those that are subject to high demand variability. In contrast to these articles, we assume that retailer efforts cannot be verified and we do not include monitoring in our paper.

Similar to our article, other researchers have also studied models concerning demand that depend on variables other than price. For instance, Desiraju and Moorthy [10] consider the role of pricing and service commitments in achieving channel coordination when the retailer is better informed about market conditions. Iyer [16] studies channel coordinating mechanisms for the manufacturer when retailers compete in price and other nonprice factors such as provision of product information, free repair, etc. However, neither of these articles include the effect of manufacturer investment in quality improving efforts.

In other work on franchise contracts, Desai and Srinivasan [9] consider the case when the franchisor has private information about demand and incurs signaling costs to credibly inform the franchisee about the demand potential. They study both linear and nonlinear price contracts and show that a two-part scheme is unable to achieve the first-best solution and therefore, propose a three-part scheme which is able to achieve the first-best profits. Although Desai and Srinivasan look at a different problem context as compared to our article, we note that the individual fixed-fee contract achieves channel coordination if the retailer is of the \( \lambda \)-type.

In another problem comparing wholesale price contracts with two-part tariff contracts, Ingene and Parry [15] show the dominance of a quantity discount wholesale price contract over two-part tariff contracts. The problem setting considered in their article is different from our article as they consider the case of two competing retailers with perfect information. In addition, their model does not include manufacturer or retailer investment in demand-influencing decisions.

The rest of the paper is organized as follows. In Section 2, we model the wholesale price contract under asymmetric cost information and determine the optimal individual and menu contracts. In Section 3, we consider two-part tariff contracts and again analyze both the individual and menu contracts. In Section 4, we compare the different contracts and determine the optimal pricing strategy for the manufacturer. Finally, we conclude and discuss future research in Section 5.

## 2. WHOLESALE PRICE CONTRACT

In this section, we consider a wholesale-price contract between a manufacturer, \( M \) selling a product through an independent retailer, \( R \). The sequence of events in the model is as follows. The manufacturer offers the product with quality \( \theta \) and wholesale price \( \omega \) to the retailer. There is no negotiation over the contract terms; the retailer may accept the contract or reject it, in which case both sides could walk away.\(^2\) If the retailer decides to accept the contract, she can influence the demand through her promotional/selling efforts, \( e \). Therefore, in addition to setting the retail price, \( p \), she would determine the optimal selling effort in response to the terms offered in the contract. The quantity ordered by the retailer is equal to the normalized demand, \( D \) (defined later).

While the retailer’s efforts may not be verifiable (and therefore cannot be contracted upon), the manufacturer anticipates the retailer’s actions and acts as a Stackelberg leader in designing the terms of the contract. Later, we consider the case of information asymmetry about the retailer’s cost of effort and model the adverse selection problem using a menu of contracts. For now, we assume that there is no information asymmetry in the model.

The normalized retail demand \( D \) is assumed to be:

\[
D = a - p + \gamma e + \lambda \theta ,
\]

where \( a \) is the market size, \( \gamma \) measures the influence of retailer’s effort on demand, and \( \lambda \) measures the impact of

\(^2\) In the rest of the paper, we implicitly assume that firms commit not to renegotiate in that if the retailer does not accept the contract terms offered by the manufacturer, there is no transaction between them.
product quality on demand. The retailer’s investment in effort has a diminishing impact on demand and the cost of effort is assumed to be \( \eta e^2 / 2 \). Similar downward sloping demand models have been used by Desai and Srinivasan [9], Desai [8], and Corbett et al. [6]. For a given contract \((\theta, \omega)\), the retailer’s objective is:

\[
\Pi_R = \max_{(\rho, \omega)} (p - \omega)[a - p + y e + \lambda \theta] - \eta e^2 / 2,
\]

which yields \( p^* = \frac{(\alpha + \lambda \theta) \eta + (\eta - \gamma^2) \omega}{(2\eta - \gamma^2)}, \quad (1) \)

\[
e^* = \frac{\gamma}{(2\eta - \gamma^2)}(\alpha + \lambda \theta - \omega). \quad (2)
\]

Define \( K = \frac{\eta}{(2\eta - \gamma^2)} \). On substituting from above and simplifying terms, we get:

\[
\Pi_R^\omega(\omega, \theta) = \frac{K}{2}(\alpha + \lambda \theta - \omega)^2, \quad (3)
\]

\[
\text{and } D(\omega, \theta) = a - p + y e + \lambda \theta = K(\alpha + \lambda \theta - \omega). \quad (4)
\]

The manufacturer invests in quality improvement efforts that may include new high-precision equipment with high reliability, fast or flexible equipment, organizational training and restructuring, etc., which improve the demand potential of the product. Another example of manufacturer-initiated effort to improve the demand potential is the investment in brand name, (see Lal [21]). By inferring the retailer’s effort reaction function in response to the terms of the contract, the manufacturer can suitably choose the contract expressions in order to maximize his profits. Let \( c \) be the manufacturer’s unit cost of production. Also, similar to the case of retailer investment in effort, the manufacturer’s cost of quality investment has diminishing impact on demand and is assumed to be \( \zeta \theta^2 / 2 \). Then, we get:

\[
\Pi_M = \max_{(\omega, \theta)} (\omega - c) D - \frac{\zeta \theta^2}{2} = K(\omega - c)(\alpha + \lambda \theta - \omega) - \frac{\zeta \theta^2}{2}.
\]

We use the superscript \((\omega)\) to denote the optimal expressions for the wholesale price contract. Solving the manufacturer’s problem above, we get:

\[
\omega^{w*} = \frac{\zeta(\alpha - c)}{(2\zeta - \lambda^2 K)} + c = \frac{\zeta \alpha + c(\zeta - \lambda^2 K)}{(2\zeta - \lambda^2 K)}, \quad (5)
\]

\[
\text{and } \theta^{w*} = \frac{\lambda K(\alpha - c)}{(2\zeta - \lambda^2 K)}. \quad (6)
\]

The manufacturer’s optimal profit is:

\[
\Pi_M^{\omega*} = \frac{\zeta K(\alpha - c)^2}{2(2\zeta - \lambda^2 K)}.
\]

From Eqs. (1–4), we get:

\[
p^{w*} = \omega^{w*} + \frac{\zeta K(\alpha - c)}{(2\zeta - \lambda^2 K)}, \quad (7)
\]

\[
e^{w*} = \frac{\gamma \zeta K(\alpha - c)}{\eta(2\zeta - \lambda^2 K)}, \quad (8)
\]

\[
D^{w*} = \frac{\zeta K(\alpha - c)}{(2\zeta - \lambda^2 K)}, \quad (9)
\]

\[
\text{and } \Pi_R^{w*} = \frac{K}{2} \left[ \frac{\zeta(\alpha - c)}{(2\zeta - \lambda^2 K)} \right]^2. \quad (10)
\]

We note from above that \( \theta^{w*}, p^{w*}, \) and \( e^{w*} \), and the profits \( \Pi_M^{w*} \) and \( \Pi_R^{w*} \) are decreasing in \( c \), as expected. Further, observe that the optimal terms of the contract offered by the manufacturer depend on the retailer’s cost of effort, \( \eta \) (which is assumed to be known to the manufacturer in this section). We can show that \((\theta^{w*}, \omega^{w*})\) and \((p^{w*}, e^{w*})\) are all decreasing in \( \eta \), as expected. That is, if the retailer has a lower cost of effort, the manufacturer provides a higher quality product and charges a higher wholesale price. The retailer then exerts more effort and is able to charge a higher retail price as well. Also, note that \( \frac{\partial \Pi_M^{w*}}{\partial \eta} = \frac{\partial \Pi_R^{w*}}{\partial \eta} < 0 \), that is, the manufacturers’ and retailer’s profits are decreasing in increasing cost of effort. The issue of information asymmetry in the cost of effort is addressed next.

2.1. Asymmetric Information on Cost of Effort

As discussed in the previous subsection, the manufacturer and the retailer profits are inversely proportional to the cost of effort. Also, if the manufacturer uses the perfect information contract, the retailer may have an incentive to misrepresent her true cost of effort. We now consider the case when the retailer’s cost of effort is not known to the manufacturer and, as such, the manufacturer may not know the retailer’s optimal choice of effort. For instance, a manufacturer selling to a first-time buyer (say, retailer) is unlikely to know the buyer’s ability to use effort to influence demand.\(^3\) The retailer could be either of two types: low-cost retailer (l-type) or high-cost retailer (h-type), between which the manufacturer cannot distinguish. However, the manufacturer has some priors regarding the distribution of retailer-type, and believes that the retailer is of l-type with probability \( r \), and of h-type with probability \((1 - r)\). Let \( \eta_l \) and \( \eta_h \) be the cost of effort for the l-type and h-type retailer, respectively, with \( \eta_h > \eta_l \).

\(^3\) In our paper, we consider the case when the retailer can observe the quality of the product and infer the manufacturer’s cost of quality. For instance, the retailer may obtain samples of the product before the contract is executed in order to judge the quality of the product. On the other hand, since retailer effort is not verifiable (and depends on the retailer-type), the cost of effort may not be known to the manufacturer.
We now consider two forms of wholesale price contracts offered by the manufacturer. In the first case, the manufacturer offers an Individual Contract (single contract) using the contract form derived for the perfect information case. Then, we consider the case where a Menu of Contracts (separate contracts) is offered with self selection, that is, the retailer correctly chooses the contract offered to her true type. A comparison of both contracts is done and we show that the menu of contracts provides higher expected profits for the manufacturer.4

2.1.1. Individual Contract

Consider the contract expressions derived for the case of perfect information [Eqs. (5) and (6)]. The manufacturer can now offer a contract \((\omega_i^w, \theta_i^w)\) using \(\eta = \eta_i\) (aimed at the \(l\)-type retailer), or a contract \((\omega_h^w, \theta_h^w)\) using \(\eta = \eta_h\) (aimed at the \(h\)-type retailer). In both cases, the retailer who is not of the true type (as assumed by the manufacturer) will also accept the contract offer if profits exceed the reservation profits.5 As such, the manufacturer’s decision would be to select the contract that maximizes his expected profits.

Define \(K_i = \frac{\eta_i}{(2\eta_i - \gamma)}\) for \(i = h, l\). Note that \(K_h, K_l \geq 0\) if \((2\eta_i - \gamma^2) \geq 0\) for \(i = h, l\). We need this condition to ensure that optimal demand, effort, etc. values are non-negative. Also, \(K_l > K_h\) if \(\eta_h > \eta_l\) (as assumed). Further, the difference \((K_l - K_h)\) increases as \(\gamma\) increases, that is, the degree of asymmetry in costs increases when retailer effort has a higher impact on demand. Suppose now, the manufacturer offers a contract designed for retailer of type \(i (i = h, l)\). If the retailer’s true type is indeed \(i\), the manufacturer’s profit will be:

\[
\Pi'_w(\omega_i^w, \theta_i^w) | R = i) = K_i (\omega_i^w - c) (\alpha + \lambda \theta_i^w - \omega_i^w) - \frac{1}{2} \xi (\theta_i^w)^2.
\]

On the other hand if the retailer type is not \(i\) but \(i^-\), the manufacturer profit is

\[
\Pi'_w(\omega_i^w, \theta_i^w) | R = i^-) = K_i^- (\omega_i^w - c) (\alpha + \lambda \theta_i^w - \omega_i^w) - \frac{1}{2} \xi (\theta_i^-)^2.
\]

where \(\theta_i^w = \frac{\lambda K_i (\alpha - c)}{2 \xi - \lambda K_i^w}\) and \(\omega_i^w = \frac{\xi (\alpha - c)}{2 \xi - \lambda K_i^w} + c\), respectively. Finally, the expected profit for the manufacturer is:

\[
\Pi'_w(\omega_i^w, \theta_i^w) = r \Pi'_w(\omega_i^w, \theta_i^w | R = l) + (1 - r) \Pi'_w(\omega_i^w, \theta_i^w | R = h),
\]

\[
= E[K_1] \left( (\alpha - c) \frac{\xi}{2 \xi - \lambda^2 K_i^w} \right)^2
- \frac{1}{2} \xi \left( (\alpha - c) \frac{\lambda K_i}{2 \xi - \lambda^2 K_i^w} \right)^2,
\]

\[
= \frac{\xi}{2} \left( \alpha - c \right)^2 \left( 2 \xi E[K_1] - \lambda^2 K_i^w \right). (11)
\]

where \(E[K_1] = r K_1 + (1 - r) K_h\). Let \(\Delta_1(r)\) denote the difference in expected manufacturer profits for contracts \((\omega_i^w, \theta_i^w)\) and \((\omega_h^w, \theta_h^w)\) for a given probability, \(r\). Then,

\[
\Delta_1(r) = \left[ \Pi'_w(\omega_i^w, \theta_i^w) \right] - \left[ \Pi'_w(\omega_h^w, \theta_h^w) \right],
\]

\[
= r \left[ \Pi'_w(\omega_i^w, \theta_i^w) | R = l \right] - \left[ \Pi'_w(\omega_h^w, \theta_h^w) | R = l \right] + (1 - r) \left[ \Pi'_w(\omega_i^w, \theta_i^w) | R = h \right] - \left[ \Pi'_w(\omega_h^w, \theta_h^w) | R = h \right]. (12)
\]

**LEMMA 1:** For the case of wholesale price individual contracts, the following hold true:

1. the manufacturer’s expected profits linearly increase in \(r\) for \(i = h, l\);
2. \(\Delta_1(r)\) strictly increases in \(r\);
3. There exists a unique \(\tilde{r}\) in \((0, 1)\) for which \(\Delta_1(\tilde{r}) = 0\).

**PROOF:** All Proofs are in Companion Appendix-A.

The result of Lemma 1 follows from the fact that for any given wholesale price contract, since the \(l\)-type retailer is more efficient and can exert more effort and generate higher demand, the profit for the manufacturer is higher as compared to the case when the retailer is of the \(h\)-type. The expected profits for the manufacturer are the weighted sum depending on the probability, \(r\). Therefore, as \(r\) increases, even if the \(h\)-type individual contract is offered, since retailers of both types will participate, the manufacturer is better off with higher values of \(r\). As shown above, Lemma 1 implies that up to a threshold probability, \(\tilde{r}\), the manufacturer is better off by offering a contract designed for the \(h\)-type retailer (comparing individual contracts only). Beyond \(\tilde{r}\), the contract designed for the \(l\)-type retailer becomes more appealing.

2.1.2. Menu of Contracts

We now design the menu of contracts \(\{ (\omega_{h1}, \theta_{h1}), (\omega_{h2}, \theta_{h2}) \}\) where \((\omega_{h1}, \theta_{h1})\) is directed to the \(l\)-type retailer and \((\omega_{h2}, \theta_{h2})\) is directed to the \(h\)-type retailer (we use ‘\(h\)’ to denote expressions for the menu contract). For the menu of contracts to

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be correctly designed, they must be self-selecting, [25]. The manufacturer’s objective is to maximize expected profits:

$$\hat{\Pi}_M^* = \max \left\{ \left( \hat{\omega}_l^w \hat{\delta}_l^w \right) \left( \hat{\omega}_h^w \hat{\delta}_h^w \right) \right\} r\left[ \hat{\Pi}_M^* (\hat{\omega}_l^w, \hat{\delta}_l^w | R = l) \right.$$ 

$$\left. + (1 - r)\hat{\Pi}_M^* (\hat{\omega}_h^w, \hat{\delta}_h^w | R = h) \right\}$$  \hspace{1cm} (13)

s.t. \hspace{1cm} $$\pi_R \left( \left( \hat{\omega}_l^w, \hat{\delta}_l^w \right) | R = l \right) \geq U,$$  \hspace{1cm} (14)

$$\pi_R \left( \left( \hat{\omega}_h^w, \hat{\delta}_h^w \right) | R = h \right) \geq U,$$  \hspace{1cm} (15)

$$\pi_R \left( \left( \hat{\omega}_l^w, \hat{\delta}_l^w \right) | R = l \right) \geq \pi_R \left( \left( \hat{\omega}_h^w, \hat{\delta}_h^w \right) | R = l \right),$$  \hspace{1cm} (16)

$$\pi_R \left( \left( \hat{\omega}_l^w, \hat{\delta}_l^w \right) | R = h \right) \geq \pi_R \left( \left( \hat{\omega}_h^w, \hat{\delta}_h^w \right) | R = h \right),$$  \hspace{1cm} (17)

where $U$ is the reservation utility of the retailer. The expected profit function of the manufacturer comprises of profit terms for the $l$-type and $h$-type retailers. Constraints (14) and (15) are the participation constraints and (16) and (17) ensure self-selection. Let

$$A = \frac{\zeta \left[ rK_l + (1 - r)K_h \right] (\alpha - c)}{2r\zeta K_i + 2(1 - r)\zeta K_h - \lambda^2 \left[ rK_i^2 + (1 - r)K_h^2 \right]},$$

\hspace{1cm} (18)

Then, the following proposition outlines the optimal solution to the manufacturer’s problem:

**PROPOSITION 1:** The solution to the Manufacturer’s Problem (13) depends on the value of the reservation utility, $U$. For $U \leq U_{\text{max}}$, where $U_{\text{max}} = \frac{K_2}{2} \left[ \frac{\zeta \left[ rK_i + (1 - r)K_h \right] (\alpha - c)}{2r\zeta K_i + 2(1 - r)\zeta K_h - \lambda^2 \left[ rK_i^2 + (1 - r)K_h^2 \right]} \right]^2$, the optimal wholesale price menu contract is as follows:

1. The optimal expressions for the $l$-type contract are:

$$\hat{\omega}_l^w = \alpha + \frac{\lambda^2 K_i A}{\zeta} - A,$$  \hspace{1cm} (19)

$$\hat{\delta}_l^w = \frac{\lambda K_i A}{\zeta}.$$  \hspace{1cm} (20)

2. The optimal expressions for the $h$-type contract are:

$$\hat{\omega}_h^w = \alpha + \frac{\lambda^2 K_h A}{\zeta} - A,$$  \hspace{1cm} (21)

$$\hat{\delta}_h^w = \frac{\lambda K_h A}{\zeta}.$$  \hspace{1cm} (22)

It is easy to show by substituting $A$ from (18) into (19) and (21) that $\hat{\omega}_l^w$ and $\hat{\omega}_h^w$ are greater than or equal to $c$, for all $r$. As we discuss in the proof, participation constraint for the $l$-type retailer (14) is ensured by the other three constraints (15)–(17) and therefore can be omitted.

There exists a threshold value of the reservation utility, $U_{\text{max}}$, such that, if $U$ is below this value, the contract offered by the manufacturer automatically satisfies the participation constraint of the $h$-type retailer (participation of the $l$-type retailer is ensured by the other constraints). However, if the reservation utility exceeds this threshold value, the manufacturer is not able to optimize his unconstrained objective function but offers the contract ensuring that the retailer accepts the contract. At some stage, if the reservation utility is very high, the manufacturer will not make any profits and will therefore not offer any contract to the retailer. In the rest of the paper, we focus on the case when the reservation utility does not exceed the threshold level and analyze the effect of $r$ on the contract selection problem for the manufacturer. The case when the reservation utility exceeds the threshold value is discussed in section 5.

On comparing the optimal expressions for the two contracts, we can see that $\hat{\omega}_l^w > \hat{\omega}_h^w$, and $\hat{\delta}_l^w > \hat{\delta}_h^w$, that is, the $l$-type retailer gets a higher quality product but has to pay a higher wholesale price to the manufacturer.

The menu of contracts yields an optimal expected profit for the manufacturer defined in (13). Therefore, as long as the optimal profit (13) exceeds the profit with the individual contract (using $\zeta = \zeta_l$ or $\zeta = \zeta_h$), the manufacturer would offer the menu of contracts. We now discuss some properties of the solution. Note that $A$ in (18) can be rewritten as:

$$A = \frac{\zeta (\alpha - c)}{2\zeta E[K] - \lambda^2 E[K^2]} E[K].$$

Then, for $i = l, h$, we have

$$\hat{\delta}_i^w = (\alpha - c) \frac{\lambda K_i E[K]}{2\zeta E[K] - \lambda^2 E[K^2]} = \frac{E[K] (2\zeta - \lambda^2 K_i)}{2\zeta E[K] - \lambda^2 E[K^2]} \hat{\delta}_i^w,$$

where $\hat{\delta}_i^w$ is the quality offered in the contract menu in Proposition 1, and $\hat{\delta}_i^w$ is the quality offered in the individual contract (from equation (6) using $K = K_i, i = l, h$).

**LEMMA 2:** For the case of wholesale price menu contract, the following hold true:

1. Product quality in both low-cost and high-cost menu contracts strictly increases in $r$;  
2. Manufacturer offers higher quality to high-cost retailer in the menu contract as compared to the quality offered in the individual contract, that is, $\hat{\delta}_h^w \geq \hat{\delta}_h^w$, for any $r$ in $[0, 1]$;
3. Manufacturer offers lower quality to low-cost retailer in the menu contract as compared to the quality offered in the individual contract, that is, $\hat{\theta}_w^l \leq \theta_w^l$, for any $r$ in $[0, 1)$.

Essentially, Lemma 2 indicates that the quality choice of the manufacturer increases in the menu contract as his belief of $l$-type retailer rises. However, the selection of quality level for different retailer types is subject to making the contract self-selecting.

2.1.3. Comparison of Wholesale Price Individual and Menu Contracts

We now compare the manufacturer’s profit for the wholesale price individual and menu contracts. In the individual contract case, the manufacturer’s expected profit can be written (from (11)) for a given $r$ and contract $i$ as:

$$\Pi_i^w(\omega_w, \theta_w) = \frac{\alpha - c}{2\xi - \lambda^2 K_i}\left(\frac{2\xi E[K] - \lambda^2 K_i^2}{\Omega_1}\right),$$

for $i = h, l$. On the other hand, the expected manufacturer profit from the menu of contracts (from (13) and Proposition 1) is:

$$E_r[\hat{\Pi}_M^w] = A \left(E[K](\alpha - c - A) + \frac{\lambda^2 AE[K^2]}{2\xi}\right)$$

$$= \frac{(\alpha - c)E[K] A}{2} = \frac{\xi(\alpha - c)^2E[K]}{2\Omega},$$

where $\Omega$ is the denominator in (18). We know that the manufacturer profits increase in $r$ in the individual contract case, and derive a similar result for the menu contracts.

**LEMMA 3:** For the case of wholesale price menu contract, the manufacturer’s expected profit is convex increasing in $r$.

This leads us to the following key result:

**PROPOSITION 2:** The manufacturer’s expected profit under wholesale price menu contract will be no less than the expected profit using wholesale price individual contracts for any $r$ in $[0, 1]$.

We observe from the results of Proposition 1 that both constraints (16) and (17) are binding at optimality, that is, the retailer would be indifferent to picking either of the contracts offered in the menu contract (designed for $l$-type or $h$-type retailer). This implies that the individual contracts can be interpreted as special instances of the menu contract satisfying both constraints (16) and (17), and consequently, individual contracts cannot outperform the menu contract.

### 3. TWO-PART TARIFF CONTRACT

The wholesale price contract derived in the previous section does not achieve channel coordination (even for the perfect information case) due to the double-marginalization effect. We now consider a two-part tariff contract where the manufacturer chooses a wholesale price and charges a transfer fee which offers the manufacturer additional flexibility.
to achieve channel coordination. To show this, we model the centralized problem where the decisions are to choose the optimal values of \( \theta, p, e \). Define:

\[
\Pi^c = \max_{(\theta, p, e)} \left[ \alpha - p + ye + \lambda \theta \right] \times (p - c) - \xi \theta^2/2 - \eta e^2/2.
\]

which yields

\[
\theta^c = \frac{K \lambda (\alpha - c)}{(\xi - \lambda^2 K)}, \quad (23)
\]

\[
p^c = c + \frac{K \xi (\alpha - c)}{(\xi - \lambda^2 K)}, \quad (24)
\]

\[
e^c = \frac{K \gamma \xi (\alpha - c)}{\eta (\xi - \lambda^2 K)}. \quad (25)
\]

Substituting in \( \Pi^c \) above, we get:

\[
\Pi^c = \frac{\xi K (\alpha - c)^2}{2(\xi - \lambda^2 K)} = B \text{(say).} \quad (26)
\]

Let us now focus our attention on the decentralized problem (we use \( f \) to denote the expressions for this contract). Now, the retailer’s problem is defined as follows:

\[
\Pi_R = \max_{(p, e, f)} \left[ \alpha - p + ye + \lambda \theta - \xi f^2/2 - F^f \right], \quad (27)
\]

which yields

\[
\Pi_R = \frac{K}{2} (\alpha + \lambda \theta - \omega f)^2 - F^f, \quad (28)
\]

when \( (\omega f, \theta f, F^f) \) is offered by the manufacturer.

On comparing \( \Pi^c \) with \( \Pi_R \) above, we note that if the manufacturer offers the contract \( (\omega f, \theta f, F^f) \), channel coordination can be achieved by setting \( \omega^c = c, \theta^c = \theta^c, \) and \( F^c = B + \xi (\theta^c)^2/2 - U \), where \( U \) is the reservation utility of the retailer. We call this as the fixed-fee contract as the manufacturer sets wholesale price equal to the marginal cost and derives all of his profits by setting the fixed-fee \( F^f \). Note that the total payment made by the retailer is \( cD + F^f \), where \( D = \alpha - p + ye + \lambda \theta \), depends on the various parameters of the model including cost of quality, effort, etc. Thus, as \( D \) increases, the unit cost, \( (cD + F^f)/D = c + F^f/D \), decreases which is similar to a quantity discount contract.\(^7\)

Since the fixed-fee contract achieves channel coordination with perfect information, we determine conditions under which fixed-fee contracts may be used by the manufacturer under asymmetric information. Later, in this section, we also consider the generalized franchise contract where wholesale price exceeds the marginal cost and the manufacturer charges a transfer fee as well.

\(^7\) We thank the Associate Editor for suggesting a quantity discount interpretation to the fixed-fee contract.

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### 3.1. Fixed-Fee Contract

Note that the fixed-fee \( F^f \) above consists of three terms; the middle term is the manufacturer’s cost of providing quality \( \theta^c \). As such, only \( B \) represents the actual profit for the manufacturer. The delineation of \( F \) into separate terms allows us to use only the “\( B \)” term in the objective function for the manufacturer.

As for the case of the wholesale price contract, we observe that the contract terms depend on \( \eta \), the retailer’s cost of effort. In fact, we can show that the transfer fee \( F^f \) (defined above) is decreasing in \( \eta \), that is, an efficient retailer (one with a lower cost of effort) would pay a higher transfer fee. Since \( K_l > K_h \), from equation (30), we see that if the manufacturer uses the perfect information contract, the \( l \)-type retailer has an incentive to misrepresent her cost of effort and make above-reservation profit. We address this issue of asymmetry in cost information next. Similar to section 2.1, the manufacturer’s priors are \( r \) and \( (1 - r) \) for the \( l \)-type and \( h \)-type retailer, respectively.

#### 3.1.1. Individual Contract

Consider the contract expressions derived for the case of perfect information [Eqs. (23)–(25)]. The manufacturer can now offer contracts (perfect information contracts) using \( \eta_l \) in one contract \( (l \)-type aimed at the low-cost retailer) or \( \eta_h \) in the other \( (h \)-type aimed at the high-cost retailer). From the perfect information case, we know that the manufacturer can set the fixed fee such that the retailer makes exactly her reservation profit.

Lets start with the case when the manufacturer offers the contract using \( \eta = \eta_l \), that is, \( (c, \theta_l, F^f) \). In this case, if the retailer is of the \( l \)-type, she would accept the contract and make the reservation profit. On the other hand, if the retailer is of the \( h \)-type, she would make less than her reservation profit and would therefore not accept the contract aimed at the \( l \)-type retailer. Then, the manufacturer’s expected profit is:

\[
\Pi_{lM}^{f_l} = r \left( \frac{\eta_l K_l (\alpha - c)^2}{2(\xi - \lambda^2 K_l)} - U \right).
\]

Note that the fixed-fee \( l \)-type individual contract is channel optimal if the retailer is of the \( l \)-type.

Now, if the manufacturer offers the contract using \( \eta = \eta_h \), that is, \( (c, \theta_h, F^f) \), if the retailer is of the \( h \)-type, she would accept the contract and make the reservation profit. On the other hand, if the retailer is of the \( l \)-type, she would also accept the contract and make greater than her reservation profit. Then, from equation (26), the manufacturer’s expected profit is:

\[
\Pi_{lM}^{f_h} = \frac{\eta_h K_h (\alpha - c)^2}{2(\xi - \lambda^2 K_h)} - U.
\]
PROPOSITION 3: For the case of fixed-fee individual contracts, there exists a unique \( \hat{r} \) in \((0, \frac{K_h}{K})\) such that the manufacturer would have higher expected profit with the l-type contract \((c, \hat{\theta}_l^f, F_l^f)\) if \( r > \hat{r} \). If \( r < \hat{r} \), the manufacturer’s expected profits would be higher with the h-type contract \((c, \hat{\theta}_h^f, F_h^f)\), where

\[
\hat{r} = \left[ \frac{\xi - \lambda^2 K_i}{\xi - \lambda^2 K_h} \left( \frac{\xi K_h (\alpha - c)^2 - 2U (\xi - \lambda^2 K_h)}{\xi K_h (\alpha - c)^2 - 2U (\xi - \lambda^2 K_h)} \right) \right].
\]

The intuition for this result is that if the manufacturer has a strong belief on the retailer being of the l-type (that is, \( r > \hat{r} \)), it is preferred to offer the contract aimed at the l-type retailer only. In this case, even though only the l-type retailer accepts the contract offer, since the fixed fee gained by the manufacturer \((F_l^f)\) is higher than the fixed fee \((F_h^f)\) derived from offering the h-type contract, the manufacturer is better off with the low-cost individual contract when \( r \) is sufficiently high.

3.1.2. Menu of Contracts

We now design the menu of contracts \( \{ (c, \hat{\theta}_l^f, F_l^f), (c, \hat{\theta}_h^f, F_h^f) \} \) where \((c, \hat{\theta}_l^f, F_l^f)\) is directed to the l-type retailer, and \((c, \hat{\theta}_h^f, F_h^f)\) is directed to the h-type retailer (again, we use \( \hat{\theta}_l^f \) and \( \hat{\theta}_h^f \) for the contract menu). From the definition of \( F_l^f \), for \( i = l, h \), once the manufacturer selects \( \hat{\theta}_l^f \) and \( \hat{B}_l^f \), the transfer fee \( \hat{F}_l^f = \hat{B}_l^f + \xi \hat{\theta}_l^f / 2 \) is determined.

Using (28) and the definitions of \( K_l \) and \( K_h \), similar to the wholesale price menu contract derived in section 2.1.2, the manufacturer’s problem is as follows:

\[
\hat{\Pi}_M = \max \left\{ \left( \hat{B}_l^f \right) \left( \hat{B}_h^f \right) \right\} \left[ r \hat{B}_l^f + (1 - r) \hat{B}_h^f \right]
\]

\[
s.t. \quad \frac{K_l}{2} \left[ \alpha + \lambda \hat{\theta}_l^f - c \right]^2 - \hat{B}_l^f - \xi \hat{\theta}_l^f / 2 \geq U,
\]

\[
\frac{K_h}{2} \left[ \alpha + \lambda \hat{\theta}_h^f - c \right]^2 - \hat{B}_h^f - \xi \hat{\theta}_h^f / 2 \geq U,
\]

\[
\frac{K_l}{2} \left[ \alpha + \lambda \hat{\theta}_l^f - c \right]^2 - \hat{B}_l^f - \xi \hat{\theta}_l^f / 2 \geq K_l / 2,
\]

\[
\times \left[ \alpha + \lambda \hat{\theta}_h^f - c \right]^2 - \hat{B}_h^f - \xi \hat{\theta}_h^f / 2 \geq \frac{K_h}{2} \times \left[ \alpha + \lambda \hat{\theta}_l^f - c \right]^2 - \hat{B}_l^f - \xi \hat{\theta}_l^f / 2 \geq \frac{K_h}{2},
\]

The optimal solution to the Manufacturer’s Problem is given in the following Proposition.

PROPOSITION 4: In the solution to the manufacturer’s problem above, the optimal fixed-fee menu contract is as follows:

1. The optimal expressions for the high-cost type contract are:

\[
\hat{\theta}_l^f = \frac{\lambda (K_h - r K_l)(\alpha - c)}{\left( \xi (1 - r) - \lambda^2 (K_h - r K_l) \right)U},
\]

\[
\hat{\theta}_h^f = \frac{2K_h \left[ \alpha + \lambda \hat{\theta}_l^f - c \right]^2 - \xi \hat{\theta}_l^f / 2 - \frac{1}{2} \times [K_l - K_h] \left[ \alpha + \lambda \hat{\theta}_h^f - c \right]^2 - \xi \hat{\theta}_h^f / 2}{U^2}. \tag{33}
\]

2. The optimal expressions for the low-cost type contract are:

\[
\hat{\theta}_l^f = \frac{\lambda K_l (\alpha - c)}{\xi (\xi - K_h \lambda^2)},
\]

\[
\hat{\theta}_h^f = \frac{K_l \left[ \alpha + \lambda \hat{\theta}_l^f - c \right]^2 - \xi \hat{\theta}_l^f / 2 - \frac{1}{2} \times [K_l - K_h] \left[ \alpha + \lambda \hat{\theta}_h^f - c \right]^2 - \xi \hat{\theta}_h^f / 2}{U^2}. \tag{34}
\]

Note from (33) that \( \hat{\theta}_l^f > 0 \) for \( r < K_h / K_l \), and zero otherwise. The quality offered for the l-type retailer in the menu contract is independent of \( r \) and is the same as the quality offered to the l-type retailer in the perfect information case. Further, from (36), we note that an increase (decrease) in \( \hat{\theta}_l^f \) would decrease (increase) the transfer fee \( \hat{B}_l^f \) for the l-type retailer. Interestingly, the manufacturer can use the quality level for one type of retailer to adjust the transfer fee charged to the other type. Similar to the results in Lemma 2, other observations are presented next:

LEMMA 4: For the case of fixed-fee menu contract, the following hold true:

1. Product quality for the low-cost retailer is independent of \( r \) and is the same as the quality offered to the low-cost retailer in the fixed-fee individual contract, that is, \( \hat{\theta}_l^f = \theta_l^f \), for any \( r \) in \((0, 1)\);

2. Product quality for the high-cost retailer is decreasing in \( r \); the manufacturer offers lower quality to the high-cost retailer in the fixed-fee contract menu as compared to the quality offered in the fixed-fee individual contract, that is, \( \hat{\theta}_h^f \leq \theta_h^f \), for any \( r \) in \((0, 1)\).

In contrast to the wholesale price menu contract, here, quality for the h-type retailer decreases in \( r \). Intuitively, as \( r \) increases, by reducing \( \hat{\theta}_l^f \), the manufacturer increases the transfer fee charged to the l-type retailer (see (36)). Similar conclusion can be derived if the spread between \( K_h \) and \( K_l \) is large. Finally, we note that for small values of \( r \) (i.e.,
As \( r \to 0 \), quality offered to the \( h \)-type retailer also approaches the centralized system solution.

Next, we analyze the expected profit for the manufacturer under this type of contract. Because of the non-negativity constraint on \( \delta_i^f \), the manufacturer profit function is defined separately for the two cases. For \( r < K_h/K_l \):

\[
\hat{\Pi}^f_M = E[\hat{\Pi}^f_M(r < K_h/K_l)] = \frac{1}{2} \zeta (\alpha - c)^2 \times \left( \frac{r K_l}{\zeta - \lambda^2 K_l} + \frac{(1-r)(K_h - r K_l)}{(1-r)\zeta - \lambda^2(K_h - r K_l)} - U \right);
\]

and, for \( r \geq K_h/K_l \),

\[
\hat{\Pi}^f_M = E[\hat{\Pi}^f_M(r \geq K_h/K_l)] = \frac{1}{2} (\alpha - c)^2 \times \left( K_h + \frac{r \lambda^2 K_l^2}{\zeta - \lambda^2 K_l} \right) - U.
\]

### 3.1.3. Comparison of Fixed-Fee Individual and Menu Contracts

In Proposition 3, we compared the manufacturer’s expected profits for the fixed-fee individual contracts. Now, we compare the fixed-fee individual contracts with the fixed-fee menu contract derived in the previous subsection.

First, observe that \( \hat{\Pi}^f_M \) is convex increasing in \( r \) for a given reservation utility of the retailer. To prove this, note that the first derivative of \( \hat{\Pi}^f_M \) wrt to \( r \) is:

\[
\frac{\partial \hat{\Pi}^f_M}{\partial r} = \frac{1}{2(\zeta - \lambda^2 K_l)} \left( \frac{\zeta (\alpha - c)(K_l - K_h)}{(1-r)\zeta - \lambda^2(K_h - r K_l)} \right)^2 > 0;
\]

The second derivative is:

\[
\frac{\partial^2 \hat{\Pi}^f_M}{\partial r^2} = \left( \frac{\zeta (\alpha - c)(K_l - K_h)}{(1-r)\zeta - \lambda^2(K_h - r K_l)} \right)^2 \times \left( \frac{1}{(1-r)\zeta - \lambda^2(K_h - r K_l)} \right) > 0 \text{ for } r < K_h/K_l.
\]

The results are given in the following two propositions.

**PROPOSITION 5:** The fixed-fee \( h \)-type individual contract is always dominated by either the fixed-fee \( l \)-type individual contract or the fixed-fee menu contract, that is, the manufacturer would never find it optimal to offer the fixed-fee individual contract for the high-cost retailer.

We know from Proposition 3 that if the probability of \( l \)-type retailer is sufficiently high, the \( l \)-type individual contract will bring more profits to the manufacturer compared to the \( h \)-type contract. If the manufacturer offers the \( h \)-type contract, retailer of both high and low cost would be willing to participate, and hence, the \( h \)-type individual contract is a special case of the menu contract and can do no better than the fixed-fee menu contract.

Since the fixed-fee \( h \)-type individual contract is always dominated, we compare the fixed-fee \( l \)-type individual contract with the fixed-fee menu contract. First, we define \( \hat{U} \) such that, \( \hat{U} = \frac{1}{2} \zeta (\alpha - c)^2 \hat{U} \), where \( \hat{U} \) is the reservation utility of the retailer.

**PROPOSITION 6:** There exists a unique \( r_i \) in \( (0, \frac{K_l}{K_h}) \) such that the manufacturer would have higher expected profit with the fixed-fee \( l \)-type individual contract if \( r > r_i \). If \( r < r_i \), the manufacturer’s expected profits would be higher with the fixed-fee menu contract, where

\[
r_i = \frac{K_l - \hat{U}(\zeta - \lambda^2 K_l)}{K_l - \hat{U}(\zeta - \lambda^2 K_l)}.
\]

From Proposition 6, we conclude that the individual contract designed for \( l \)-type retailer yields better expected profits for the manufacturer if and only if \( r > r_i \). Otherwise, the menu contract outperforms the individual low-cost contract. Essentially, when the manufacturer has a high prior on the retailer being of the low-type, it may be optimal not to distinguish between the retailer types and lose some efficiency by offering the menu contract. From Proposition 4 we know that the efficiency loss is due to the fact that under fixed-fee menu contract, the \( l \)-type contract must be designed to prevent the \( l \)-type retailer from choosing the \( h \)-type contract. Consequently, at optimality, we show that constraint (31) is binding which limits the manufacturer’s pricing and quality choices. As such, the manufacturer incurs a cost for screening, which is needed to separate the retailer types. Clearly, the optimal manufacturer profits from selling to a certain type of retailer are always higher under perfect information. The gap between profits can be interpreted as the fee that the manufacturer needs to pay to learn the true type of the retailer. Above result indicates that when the probability of \( l \)-type retailer is sufficiently high, it is not worthwhile for the manufacturer to screen retailers and to pay this fee. Hence, for large values of \( r \), \( l \)-type individual contract is preferable to the manufacturer. The results are illustrated in Fig. 2. On the flip side, we note that the \( h \)-type individual contract is never preferable to the manufacturer even when \( r \) is small. The \( h \)-type individual contract always generates lower profit for the manufacturer compared to the \( l \)-type individual contract. As such the contract design must include the \( l \)-type retailer for any positive \( r \) value.

The decision to offer the low-cost individual or menu contract depends on the probability \( r_i \). We now derive some
bounds on the values of \( r_t \) which will be useful in deriving the subsequent results in the next section.

**COROLLARY 1:** The critical threshold probability \( r_t \) is a decreasing function of \( \zeta \) for a given \( U_r \) and a decreasing function of \( \hat{U} \) (or \( U \)) for a given \( \zeta \). Also, the bounds are:

\[
\frac{k_h}{k_l} \geq r_t \geq \frac{1}{4k_l-k_h}.
\]

In Appendix B, we consider the fixed-fee pooling contract and show that is dominated by the menu contract.

### 3.2. General Franchise Contract

In this section, we consider the general franchise contract where the manufacturer sets the wholesale price (at a level not necessarily equal to the marginal cost) in addition to the fixed fee charged to the retailer. Specifically, we consider two forms of franchise contracts: menu contract and pooling contract. We do not have to consider the franchise individual contract as it is the same as the fixed-fee individual contract (if there is no information asymmetry, the optimal franchise contract is essentially a fixed-fee contract only as wholesale price is set equal to production cost). In the analysis below, the franchise menu contract is derived whereas the franchise pooling contract is given in Appendix B and is shown to be dominated by the menu contract.

Similar to the fixed-fee menu contract in section 3.1.2, we design the franchise menu contract \( \{\hat{o}_F^t, \hat{\theta}_F^t, \hat{F}_F^t\}, \{\hat{o}_h^t, \hat{\theta}_h^t, \hat{F}_h^t\} \) directed to the \( t \)-type and \( h \)-type retailer, respectively. We use the superscript \( F \) for the expressions in this contract.

The manufacturer’s problem is as follows:

\[
\hat{\Pi}_M^F = \max \left\{ \left( \hat{o}_F^t \hat{\theta}_F^t \hat{F}_F^t \right) \left( \hat{o}_h^t \hat{\theta}_h^t \hat{F}_h^t \right) \right\} \\
\times \left[ r \left\{ \left( \hat{o}_h^t - c \right) K_l \left( \alpha + \lambda \hat{o}_F^h - \hat{\theta}_F^h \right) - \frac{\xi \hat{\theta}_F^h}{2} + \hat{F}_F^h \right\} \\
+ \left( 1 - r \right) \left\{ \left( \hat{o}_F^h - c \right) K_h \left( \alpha + \lambda \hat{o}_F^h - \hat{\theta}_F^h \right) \\
- \frac{\xi \hat{\theta}_F^h}{2} + \hat{F}_F^h \right\} \right]\right\}
\]

s.t.

\[
\frac{K_l}{2} \left[ \left( \alpha + \lambda \hat{o}_F^h - \hat{\theta}_F^h \right)^2 - \hat{F}_F^h \right] - \hat{F}_F^h \geq U \cdot fr, (38)
\]

\[
\frac{K_h}{2} \left[ \left( \alpha + \lambda \hat{o}_F^h - \hat{\theta}_F^h \right)^2 - \hat{F}_F^h \right] \geq \frac{K_h}{2}
\]

\[
\frac{K_l}{2} \left[ \left( \alpha + \lambda \hat{o}_F^h - \hat{\theta}_F^h \right)^2 - \hat{F}_F^h \right] - \hat{F}_F^h \geq \frac{K_h}{2}
\]

The optimal solution to the manufacturer’s problem is given in the following proposition.

**PROPOSITION 7:** In the solution to the manufacturer’s problem (38), the optimal franchise menu contract is as follows:

- **The optimal expressions for the high-cost type contract are:**
  \[
  \hat{\omega}_h^F = c + \frac{(\alpha - c) \xi (K_l - K_h)}{r \xi (K_l - K_h) + (1 - r) K_h (\zeta - \lambda^2 K_h)},
  \]
  \[
  \hat{\theta}_h^F = \frac{(1 - r) \lambda K_h^2 (\alpha - c)}{r \xi (K_l - K_h) + (1 - r) K_h (\zeta - \lambda^2 K_h)},
  \]
  \[
  \hat{F}_h^F = \frac{K_h}{2} \left[ \left( \alpha + \lambda \hat{o}_F^h - \hat{\theta}_F^h \right)^2 - \hat{F}_F^h \right] - U.
  \]

- **The optimal expressions for the low-cost type contract are:**
  \[
  \hat{\omega}_t^F = c,
  \]
  \[
  \hat{\theta}_t^F = \frac{\lambda K_l (\alpha - c)}{\xi (K_l - K_h)},
  \]
  \[
  \hat{F}_t^F = \frac{K_l}{2} \left[ \left( \alpha + \lambda \hat{o}_F^h - \hat{\theta}_F^h \right)^2 - \frac{1}{2} (K_l - K_h) \right]
  \]
  \[
  \times \left[ \left( \alpha + \lambda \hat{o}_F^h - \hat{\theta}_F^h \right)^2 - U \right].
  \]

We note that the franchise menu contract is similar to the fixed-fee menu contract derived in section 3.1.2. While the
l-type retailer gets wholesale price equal to the marginal cost, the contract for the h-type retailer includes a wholesale price that strictly exceeds marginal cost, that is, the manufacturer fully utilizes the two-part tariff for the h-type retailer. Finally, as shown in Appendix B, the franchise pooling contract is dominated by the menu contract.

4. COMPARISON OF CONTRACT TYPES

In sections 2 and 3, we analyzed wholesale price and two-part tariff contracts, respectively, and showed that while the wholesale menu contract dominated other forms of wholesale contracts, for the fixed-fee contract, both the l-type individual or menu contract may be offered by the manufacturer. We now compare the different contract types in this section.

4.1. Comparison Between Wholesale Price and Franchise Contracts

Note that the wholesale price menu contract (in section 2.1.2) is a special case of the general franchise menu contract with \( \hat{F}_{bl} = \hat{F}_{bl} = 0 \). First, we show that the franchise menu contract dominates the wholesale price menu contract, that is, the optimal solution to the franchise menu contract will not have both \( \hat{F}_{bl} = \hat{F}_{bl} = 0 \).

PROPOSITION 8: It is never optimal for the manufacturer to set transfer fees, \( \hat{F}_{bl} = \hat{F}_{bl} = 0 \), that is, the wholesale price menu contract is always dominated by the franchise menu contract.

The above result establishes that the wholesale price menu contract, which is a special case of the general franchise menu contract (when transfer fees are set to zero), will not be offered as the franchise contract yields higher profits for the manufacturer. Essentially, as the proof indicates, it is never optimal to set zero transfer fees in the contract. It is however important to note that even though the wholesale menu contract is dominated, we need the results derived in Section 2 (that individual wholesale price contracts are dominated by the franchise menu contract) in order to eliminate the wholesale price only contract.

4.2. Comparison Between Fixed-Fee and General Franchise Contracts

While the wholesale price contract is dominated by the franchise menu contract, would the fixed-fee contract, which is channel optimal under perfect information, be preferred to the general franchise menu contract in the imperfect information case? To analyze this, we compare the expected profits for the manufacturer for the two contracts.

From (38), the expected profit for the manufacturer in the franchise menu contract is:

\[
\hat{\Pi}_{M}^{F} = r \left( \frac{(\alpha - c)^{2} \zeta K_{l}}{2(\zeta - \lambda^{2} K_{l})} + (1 - r)^{2} \right) \times \frac{(\alpha - c)^{2} \zeta K_{h}^{2}}{2(r \zeta (K_{l} - K_{h}) + (1 - r) K_{h}(\zeta - \lambda^{2} K_{h}))} - U
\]

The expected profit for the manufacturer by using the fixed-fee low-cost individual contract is given in section 3.1.1.

As shown in the following proposition, there exist conditions such that the manufacturer would prefer the Individual fixed-fee contract over the more general franchise menu contract. As noted earlier in section 3.1.3, we define \( U \) such that, \( U = \frac{1}{2} \zeta (\alpha - c)^{2} \hat{U} \).

PROPOSITION 9:

1. The manufacturer would offer the general franchise menu contract if \( r \leq r_{f} \);
2. The manufacturer would offer the fixed-fee l-type individual contract if \( r > r_{f} \),

where \( r_{f} = \frac{K_{h}^{2} - \hat{U} K_{h}(\zeta - \lambda^{2} K_{h})}{K_{h}^{2} - \hat{U}(K_{h}(\zeta - \lambda^{2} K_{h}) - \zeta (K_{l} - K_{h}))} \).

It is easy to note that \( r_{f} < 1 \) since \( K_{l} > K_{h} \). Also, non-negativity of \( r_{f} \) is ensured since the maximum reservation utility of the retailer is less than the centralized system.

Figure 3. Comparison of fixed-fee l-type individual and franchise menu contracts. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]
profit for the $h$-type retailer case. The result is illustrated in Fig. 3. A simple comparison between (37) and (50) will reveal that $r_f \geq r_i$ implying that the threshold probability between the fixed-fee $l$-type individual contract and the general franchise contract is no less than the threshold probability between the former contract and the fixed-fee menu contract. This is intuitive as the general franchise contract provides more flexibility to the manufacturer compared to the fixed-fee menu contract.

COROLLARY 2: The manufacturer is more likely to offer the fixed-fee $l$-type individual contract (as compared to the franchise menu contract) to a retailer with high reservation utility since $r_f$ decreases in $\hat{U}$. Also, preference for the fixed-fee contract is stronger when the difference in retailer costs (measured in terms of the difference, $K_l - K_h$) is higher.

Essentially, we show that when the manufacturer’s prior on the retailer being of the $l$-type exceeds a certain threshold, it may be preferable to offer the fixed-fee $l$-type individual contract as compared to the franchise menu contract. Then, the manufacturer would exclude the $h$-type retailer and offer an individual contract that is accepted by the $l$-type retailer only. Intuitively, the individual contract is efficient for the case of perfect information and as such, the manufacturer may prefer it to the franchise menu contract (which has some loss in efficiency in order to make it self-selecting for both retailer types) when the likelihood of the retailer being of the $l$-type is high. Further, we also note that the preference for the Individual fixed-fee contract increases when the reservation utility of the retailer and the degree of information asymmetry in costs is high. As such, the manufacturer becomes more willing to exclude the $h$-type retailer by offering the fixed-fee individual contract which is only accepted by the $l$-type retailer.

5. CONCLUSIONS AND FUTURE RESEARCH

In this paper, we examine and compare different contracting approaches that a manufacturer may adopt selling to a retailer when end-customer demand depends on the manufacturer’s chosen quality and the retailer’s selling effort. It has been shown in the literature that franchising offers more flexibility to the manufacturer as compared to price-only contracts. Nonetheless, both approaches are common in practice. In our analysis, we examine whether wholesale contracts might indeed be preferable to the manufacturer when a contract design requires additional non-monetary terms such as product quality. Our results indicate that while the general franchise menu contract dominates the wholesale menu contract, the manufacturer may, at times, prefer to use the fixed-fee contract as opposed to the general franchise menu contract.

The analytical results derived in the paper are for the case when the retailer’s reservation utility is less than a threshold value ($U_{\text{max}}$). While we do not derive analytical results for the case when $U \geq U_{\text{max}}$, in our numerical analysis, we observed that as the reservation utility increases up to point beyond $U_{\text{max}}$, the difference between manufacturer profits for fixed-fee and wholesale contracts continues to decrease and the results are the same as derived in the paper. Beyond a certain level, if the reservation profit continues to increase, fixed-fee contracts will outperform the wholesale contracts since the expected profits for the manufacturer will also decrease under wholesale contracts. For sufficiently high reservation profits, the manufacturer will ignore the $h$-type retailer and design fixed-fee contracts for the $l$-type retailer only. Finally, if the reservation profit is too large, the manufacturer may refuse to offer any contract at all.

In this article, we considered the case when there is no demand uncertainty and we were able to derive interesting managerial insights into the contract selection problem for the manufacturer. In future research, we plan to generalize the model in order to study the effect of demand variability. In addition, if the manufacturer’s quality is not observed at the time of signing the contract, we encounter a double moral hazard problem. Other possible extensions include a menu of products offered by the manufacturer with different moral hazard problem. Other possible extensions include a menu of products offered by the manufacturer with different

APPENDIX

Appendix A: Companion

PROOF OF LEMMA 1: 1. For a given contract type $(w_i^h, \theta_i^h)$, $i = h, l$, the marginal change in manufacturer’s expected profits with respect to $r$ is

$$\frac{\partial \Pi_{w}^{h}(w_i^h, \theta_i^h)}{\partial r} = (K_l - K_h) \left( \frac{\alpha - \zeta}{2\zeta - \lambda^2K_l} \right)^2 > 0,$$

since $K_l > K_h$. Also, the first derivative is independent of $r$. Therefore, the manufacturer profits linearly increase in $r$ for either type of Individual Contract offered.

2. The derivative of $\Delta_1(r)$ with respect to $r$ can be written as:

$$\frac{\partial \Delta_1(r)}{\partial r} = (\Pi_{w}^{h}(w_i^h, \theta_i^h)|R = l) - \Pi_{w}^{h}(w_i^h, \theta_i^h)|R = l) + (\Pi_{w}^{h}(w_i, \theta_i)|R = h) - \Pi_{w}^{h}(w_i, \theta_i)|R = h) > 0,$$

since for a given retailer type $i$, contract $(w_i, \theta_i)$ maximizes the manufacturer profits. Hence, $\Delta_1(r)$ increases in $r$.

3. From 1) and 2) above,

$$\Delta_1(0) = \Pi_{w}^{h}(w_i, \theta_i)|R = h) - \Pi_{w}^{h}(w_i, \theta_i)|R = h) < 0,$$

and $\Delta_1(1) = \Pi_{w}^{h}(w_i, \theta_i)|R = l) - \Pi_{w}^{h}(w_i, \theta_i)|R = l) > 0.$
Since $\Delta_1$ strictly increases in $r$, there must exist a $\tau$, such that $0 < \tau < 1$, and $\Delta_1(\tau) = 0$. □

**PROOF OF PROPOSITION 1:** Using (3) and the definitions of $K_l, K_h$, we can write constraints (14)–(17) as:

\[
\begin{align*}
\frac{K_1}{2} [a + \lambda \hat{h}_w - \hat{q}_w^2] &\geq U, \\
\frac{K_2}{2} [a + \lambda \hat{h}_w - \hat{q}_w^2] &\geq U, \\
\frac{K_1}{2} [a + \lambda \hat{h}_w - \hat{q}_w^2] &\geq \frac{K_1}{2} [a + \lambda \hat{h}_w - \hat{q}_w^2], \\
\frac{K_1}{2} [a + \lambda \hat{h}_w - \hat{q}_w^2] &\geq \frac{K_2}{2} [a + \lambda \hat{h}_w - \hat{q}_w^2].
\end{align*}
\]

(A1) (A2) (A3) (A4)

We note that constraints in (A3) and (A4) imply that $(a + \lambda \hat{h}_w - \hat{q}_w^2) = (a + \lambda \hat{h}_w - \hat{q}_w^2)$. Hence, we can replace the last two constraints with this equality. It appears that both retailers may be indifferent to the contract offered to them and as such the menu contract does not distinguish between different types of retailers. However, suitable use of $\varepsilon$ (with $\varepsilon \rightarrow 0$) in the RHS of (A3) and in the LHS of (A4) can ensure that a separating contract can be achieved. We first ignore the first two constraints in (A1) and (A2), and solve the optimization problem. Later we note that the optimal solution to the relaxed problem does satisfy the constraints in (A1) and (A2) when $U \leq U_{\text{max}}$, as assumed in the statement of the Proposition.

From (13), we can write the manufacturer’s profits as:

\[
\Pi_M^H [\hat{q}_w^*, \hat{\theta}_w, \hat{\mu} | R = l] = (\hat{q}_w^* - c) K_l (a + \lambda \hat{h}_w - \hat{q}_w^2)^{-1} - \lambda \hat{q}_w^2 / 2, \quad (A5)
\]

\[
\Pi_M^H [\hat{q}_w^*, \hat{\theta}_w | R = h] = (\hat{q}_w^* - c) K_h (a + \lambda \hat{h}_w - \hat{q}_w^2) - \lambda \hat{q}_w^2 / 2. \quad (A6)
\]

Using Lagrangean relaxation and substituting from (A5) and (A6) in (13), we get:

\[
\hat{\Pi}_M^H = \max \{ \hat{q}_w^* \geq 0, \hat{\theta}_w \geq 0, \hat{\mu} \geq 0 \} \left[ K_l (\hat{q}_w^* - c) \right. \\
\left. \times (a + \lambda \hat{h}_w - \hat{q}_w^2) - \lambda \hat{q}_w^2 / 2 \right] + (1 - r) \left[ K_h (\hat{q}_w^* - c) K_h (a + \lambda \hat{h}_w - \hat{q}_w^2) - \lambda \hat{q}_w^2 / 2 \right] + \mu \left[ \hat{\theta}_w - \hat{\theta}_w^* \right],
\]

where $\mu$ is the Lagrange multiplier. First order optimality conditions yield the following five equations:

\[
\frac{\partial \hat{\Pi}_M^H}{\partial \hat{\theta}_w} = r K_l (a + \lambda \hat{h}_w + c - 2\hat{q}_w^2) + \mu = 0 \quad (A7)
\]

\[
\frac{\partial \hat{\Pi}_M^H}{\partial \hat{q}_w^2} = (1-r) K_l (a + \lambda \hat{h}_w + c - 2\hat{q}_w^2) - \mu = 0 \quad (A8)
\]

\[
\frac{\partial \hat{\Pi}_M^H}{\partial \hat{\theta}_w} = \left( \lambda K_l (\hat{q}_w^* - c) - \lambda \right) - \lambda \mu = 0 \quad (A9)
\]

\[
\frac{\partial \hat{\Pi}_M^H}{\partial \hat{q}_w^2} = \left( \lambda K_h (\hat{q}_w^* - c) - \lambda \right) + \lambda \mu = 0 \quad (A10)
\]

\[
\frac{\partial \hat{\Pi}_M^H}{\partial \mu} = \left( \hat{\theta}_w - \hat{\theta}_w^* \right) - \lambda (\hat{\theta}_w - \hat{\theta}_w^*) = 0 \quad (A11)
\]

Solving equations (A7–A11) for $\hat{q}_w^*, \hat{\theta}_w^*, \hat{\mu}$, and $\lambda$, we get (19), (20), (21), and (22). Finally, note that the left hand side in (A2) is less than or equal to $U_{\text{max}}$. Since by assumption $U \leq U_{\text{max}}$, the constraint must hold. Moreover, since $\eta_l > \eta_r$, constraint (A2) also implies (A1). In fact, constraint (A1) is a strict inequality, that is, the low-cost retailer makes profits in excess of the reservation profit. □

**PROOF OF LEMMA 2:** i. It is sufficient to show that the derivative of quality function with respect to $r$ is positive so as to complete the proof of this part. Let $\Delta$ represent the denominator in (18). We can write the derivative of $A$ with respect to $r$ as follows:

\[
A' (r) = \frac{\lambda^2 (K_l - K_h) K_l K_h}{\Omega^2} > 0
\]

Hence, since $A$ increases in $r$, quality for any $i$ must be increasing with $r$ as well.

ii. To see that menu contracts offer higher quality to high-cost retailer, note from (18) that for $r > 0$, $\hat{\theta}_w^* = \hat{\theta}_w^*$. From first part of the proof we know that $\hat{\theta}_w^*$ increases in $r$ and that $\hat{\theta}_w^*$ is independent of $r$. Thus for $r \geq 0$, $\hat{\theta}_w^* \geq \theta_w^*$.

iii. Observe from (18) that for $r = 1$, $\hat{\theta}_w^* = \theta_w^*$. Using a similar approach as in part ii), we can show that for $r \leq 1$, $\hat{\theta}_w^* \leq \theta_w^*$. □

**PROOF OF LEMMA 3:** The derivative of $E_i [\hat{\Pi}_M^H]$ is as follows:

\[
\frac{\partial E_i [\hat{\Pi}_M^H]}{\partial r} = \frac{1}{2} \xi (a - c)^2 (K_l - K_h) A + \frac{(a - c) E_i [K] A'}{\Omega} > 0
\]

since $K_l > K_h$ and $A' > 0$. Hence, we show that manufacturer profits increase in $r$. However, the increase is not linear as compared to the individual contract case. To observe convexity, we compute the second order derivative:

\[
\frac{\partial^2 E_i [\hat{\Pi}_M^H]}{\partial r^2} = \zeta (a - c)^2 (K_l - K_h)^2 \left( \Omega - E_i [K] \Omega / \Omega^2 \right) > 0
\]

where $\Omega$ is derivative of $\Omega$ with respect to $r$. Thus, the above function returns a non-negative value implying that the manufacturer’s profit function is convex in $r$. □

**PROOF OF PROPOSITION 2:** First, we consider the difference in the expected profits using the menu contract and high-cost individual contract $(\theta_w^*, \hat{\theta}_w^*)$. Define,

\[
\Delta_2 (r) = E_i [\hat{\Pi}_M^H - \Pi_i^H (\theta_w^*, \hat{\theta}_w^*)]
\]

From Lemmas 1 and 3, notice that $\Delta_2 (r)$ is convex in $r$. Let $\Delta_2 (r)$ denote the first order derivative of $\Delta_2 (r)$ with respect to $r$. Observe that at $r = 0$, $\Delta_2 (0) = 0$ and,

\[
\Delta_2 (0) = \frac{1}{2} \xi (a - c)^2 (K_l - K_h)^2 \frac{\lambda^2 K_h}{(2 \xi - \lambda^2 K_h)^2} > 0
\]

Since $\Delta_2 (r)$ is convex and increasing at $r = 0$, $\Delta_2 (r)$ must be positive for any $r > 0$.

Now, consider the low-cost individual contract $(\theta_w^*, \hat{\theta}_w^*)$. Let $\Delta_3 (r) = E_i [\hat{\Pi}_M^H - \Pi_i^H (\theta_w^*, \hat{\theta}_w^*)]$, where $\Delta_3 (r)$ is also convex in $r$. Observe that $\Delta_3 (1) = 0$, and

\[
\Delta_3 (1) = -\frac{1}{2} \xi (a - c)^2 (K_l - K_h)^2 \left( \frac{\lambda^2 K_h}{(2 \xi - \lambda^2 K_h)^2} \right) < 0
\]

implying that the difference is decreasing at $r = 1$ and because of convexity it must be decreasing for any $r < 0$ as well. Thus, $\Delta_3 (r) \geq 0$ for $r \leq 1$. □

**PROOF OF PROPOSITION 3:** The proof follows from comparing the manufacturer’s expected profits $\Pi_M^H$ and $\Pi_M^H$. It is easy to verify that $\hat{r}$ is
less than $\frac{K_b}{\bar{r}}$. Note that if $U = 0$, we get $\hat{r} = \frac{K_b}{K_l} \left( \frac{-\lambda K_l}{-\lambda + K_l} \right)$ which is also less than $\frac{K_b}{\bar{r}}$. □

**PROOF OF PROPOSITION 4:** Let $\mu_1, \mu_2, \mu_3,$ and $\mu_4$ be the Lagrangian multipliers for the four constraints (29–32). Then, we can write the Lagrangian as $L = \Pi_M + \mu_1 \cdot \Pi_2 + \mu_3 \cdot \Pi_3 + \mu_4 \cdot \Pi_4$. Taking derivatives, we get:

$$\frac{\partial L}{\partial B_1} = 0 \Rightarrow \mu_1 - \mu_3 + \mu_4 = 0, \quad (A12)$$

$$\frac{\partial L}{\partial B_1} = 0 \Rightarrow (1 - r) - \mu_2 + \mu_3 - \mu_4 = 0. \quad (A13)$$

Adding the foc’s, we get $\mu_1 + \mu_2 = 1$, that is, at least one of constraints (29) or (30) is activated. There exist three possible scenarios here: (i) $\mu_1 > 0$ only; (ii) $\mu_2 > 0$ only; (iii) both $\mu_1$ and $\mu_2 > 0$. We can see that if both $\mu_1$ and $\mu_2 > 0$, that is, both constraints (29) and (30) are activated, we get a contradiction from the other two conditions (31) and (32). The same is true if only $\mu_1 > 0$. Therefore, we have $\mu_1 = 0$ and $\mu_2 = 1$. Substituting in (A12) and (A13), we get $\mu_3 - \mu_4 = r$, which implies that $\mu_3 > 0$. Again, we can see that if $\mu_4 > 0$ as well, we have a contradiction from (31) and (32). In summary, we have $\mu_2 > 0$ and $\mu_3 = \mu_4 = 0$. Therefore, constraints (30) and (31) are activated. On rearranging terms, we get

$$\hat{B}_1 = \frac{K_b}{2} \left( \alpha + \lambda \hat{B}_1 - c \right)^2 - \frac{\hat{B}_1^2}{2} - \lambda U. \quad (A14)$$

On substituting $\hat{B}_1$ and $\hat{B}_1$ from above and taking partial derivatives and equating to zero, we get (33) (non-negativity of $\hat{B}_1^2$ is ensured by using the maximum of zero and the solution using the foc), (34) and (35), (36). □

**PROOF OF LEMMA 4:** The first part of the proof is from straightforward comparison of (35) with (23) (using $K = K_l$). For the second part, the derivative of $\hat{B}_1^2$ wrt $r$ is:

$$\frac{\partial \hat{B}_1^2}{\partial r} = \frac{\lambda (\alpha - c) \zeta (K_l - K_b)}{(\Omega^2)^2} < 0,$$

where $\Omega^2$ is the denominator in (33). From (33), we also observe that for $r \geq K_b/K_l$, $\hat{B}_1^2 = 0$. Finally, note that $\hat{B}_1^2 |_{r=0} = \hat{B}_1^2$, but since $\hat{B}_1$ is decreasing in $r$, we have $\hat{B}_1^2 \leq \hat{B}_1^2$. □

**PROOF OF PROPOSITION 5:** We note from Proposition 3 that the high-cost individual franchise contract is dominated by the low-cost individual contract for $r > \bar{r}$. Therefore, we only need to consider the case when $r \leq \bar{r}$. Since $\bar{r} \leq K_b/K_l$, we need to compare $\hat{B}_1$ with $\hat{B}_1$.

Note that $\hat{B}_1 |_{r=0} = \frac{\lambda (\alpha - c) \zeta (K_l - K_b)}{2(\lambda - K_l)} - \frac{U}{\hat{B}_1} = \hat{B}_1$ (which is independent of $r$). But since $\hat{B}_1^2$ is increasing in $r$, we have $\hat{B}_1^2 \geq \hat{B}_1^2$ for $r \leq \bar{r}$. □

**PROOF OF PROPOSITION 6:** We can show that $r_1$ (defined above) is less than $K_b/K_l$. Therefore, we consider the case when $r \leq K_b/K_l$. Now, using $\hat{U}$, we can rewrite the profit functions for the manufacturer as:

$$\hat{\Pi}_M^f = \frac{1}{2} \xi (\alpha - c)^2 \left( \frac{r K_l}{\xi - \lambda^2 K_l} + \frac{(1 - r) (K_b - r K_l)}{(1 - r) \xi - \lambda^2 (K_b - r K_l)} - \hat{U} \right),$$

and $\hat{\Pi}_M^{f_j} = \frac{1}{2} \xi (\alpha - c)^2 \left( \frac{K_l}{\xi - \lambda^2 K_l} - \hat{U} \right)$.

Define $\Delta_1(r) = \hat{\Pi}_M^f - \hat{\Pi}_M^{f_j} = \frac{1}{2} \xi (\alpha - c)^2 (1 - r)$

$$\times \left( \frac{(K_b - r K_l)}{(1 - r) \xi - \lambda^2 (K_b - r K_l)} - \hat{U} \right).$$

Note that the term within the parenthesis of $\Delta_1(r)$ is decreasing in $r$ and is equal to zero for $r = r_1$. Therefore, for $r > r_1$, $\Delta_1 < 0$.

For $r > K_b/K_l$, define,

$$\Delta_2 = \hat{\Pi}_M^f - \hat{\Pi}_M^{f_j} = \frac{1}{2} \xi (\alpha - c)^2 \left( K_b + \frac{r K_l^2}{\xi - \lambda^2 K_l} - \frac{r K_l \xi}{\xi - \lambda^2 K_l} \right)$$

$-(1 - r) U,$

$$= \frac{1}{2} (\alpha - c)^2 (K_b - r K_l) - (1 - r) U < 0.$$ □

**PROOF OF COROLLARY 1:** For a given $\hat{U}$, the first derivative of $r_1$ wrt $\xi$ is:

$$\frac{\partial r_1}{\partial \xi} = \frac{\lambda (K_b - K_l)(1 + \hat{U})^2}{(K_l \xi - \lambda^2 K_l)^2} < 0.$$ Similarity, we can easily show that $r_1$ decreases in $\hat{U}$ for a given $\xi$. Therefore, the maximum value for $r_1$ is achieved when $\hat{U} = 0$, that is, $r_1 |_{r=0} = \frac{K_b}{K_l}$. Now, we note that the reservation utility for the retailer is less than equal to $\hat{U}_{\text{max}}$ as defined in Proposition 1. Further, corresponding to the maximum reservation utility $\hat{U}_{\text{max}}$, we get $\hat{U}_{\text{max}}$. Since $r_1$ is a decreasing function in $\hat{U}$ and $\xi$, we substitute the highest value $\hat{U}_{\text{max}}$ and take the limit as $\xi \to \infty$, and get:

$$\lim_{\xi \to \infty} r_1 = \frac{3K_b}{4K_l - K_b},$$

which gives the minimum possible value that $r_1$ can take. □

**PROOF OF PROPOSITION 7:** The proof is similar to that of Proposition 4. □

**PROOF OF PROPOSITION 8:** Let $\mu_1, \mu_2, \mu_3$, and $\mu_4$ be the Lagrangian multipliers for the four constraints (39–42). Also, let $\mu_5$ and $\mu_6$ be the Lagrangian multipliers on the negativity constraints for $\hat{F}_l^f$ and $\hat{F}_h^f$, respectively. Then, using (38), we can write the Lagrangian as $L = \Pi_M + \mu_1 \cdot \Pi_2 + \mu_2 \cdot \Pi_3 + \mu_3 \cdot \Pi_4 + \mu_4 \cdot \Pi_5 + \mu_5 \cdot \Pi_6$. Taking derivatives, we get:

$$\frac{\partial L}{\partial \hat{F}_l} = 0 \Rightarrow r - \mu_1 - \mu_3 + \mu_4 + \mu_5 = 0, \quad (A16)$$

$$\frac{\partial L}{\partial \hat{F}_h} = 0 \Rightarrow (1 - r) - \mu_2 + \mu_3 - \mu_4 + \mu_6 = 0. \quad (A17)$$

Adding the foc’s, we get $\mu_1 + \mu_2 = 1 + \mu_3 + \mu_6$, that is, at least one of constraints (39) or (40) is activated. Also, from (A16), at least one of $\mu_1$ and $\mu_3 > 0$. We can easily show that $\mu_1 > 0$ results in a contradiction (since $K_l > K_b$), and therefore, $\mu_2 > 0$ and $\mu_3 > 0$.

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Now suppose that \( \hat{L}^f = \hat{L}^h = 0 \). Then, we note that constraint (42) must also be binding, and \( \alpha + \lambda \hat{\beta}_f - \hat{\alpha}_h = \alpha + \hat{\lambda} \hat{\beta}_h - \hat{\alpha}_h \). Now

\[
\frac{\partial L}{\partial \hat{\alpha}_h} = -\hat{r}(\hat{\alpha}_h - c)K_l + (rK_l - \mu_3 K_l + \mu_4 K_h)(\alpha + \hat{\lambda} \hat{\beta}_h - \hat{\alpha}_h) = 0.
\]

Since \( \mu_3 = r + \mu_4 + \mu_5 \), we get \( \hat{\alpha}_h \leq c \) since \( K_l > K_h \).

Also, solving for \( \hat{\beta}_f \), we get \( \hat{\alpha}_h = \frac{\hat{r}K_l(\alpha - \hat{\alpha}_h)}{(\hat{\zeta} - 2\hat{\alpha}_h)} \). Then, we get \( \hat{\alpha}_h = \hat{\alpha}_h - \hat{\alpha}_h \).

From (40), we get

\[
\frac{K_h}{2} (\alpha + \lambda \hat{\beta}_f - \hat{\alpha}_h) = \frac{K_h}{2} (\alpha - \hat{\alpha}_h)^2 \frac{\hat{\zeta}^2}{(\hat{\zeta} - 2\hat{\alpha}_h)^2} = U.
\]

Since \( \hat{\alpha}_h \leq c \), on simplification, we get

\[
\frac{U}{k} \geq \frac{K_h}{2} (\alpha - c)^2 \frac{\hat{\zeta}^2}{(\hat{\zeta} - 2\hat{\alpha}_h)^2} > \frac{\hat{K}_h \zeta (\alpha - c)^2}{2(\hat{\zeta} - 2\hat{\alpha}_h)^2},
\]

which is a contradiction since the right hand side above is the centralized system profit when the retailer is of the high-type. Clearly, we need the retailer’s reservation utility to be less than the centralized system profit or less the manufacturer would choose not to do business with the retailer. □

PROOF OF PROPOSITION 9: We have

\[
\hat{\Pi}_M - \Pi_{M}^{\hat{f}} = (1 - r)(\alpha - c)^2 \frac{\hat{\zeta}^2}{2} \left[ 1 - \frac{1}{r} \frac{K_h^2}{\hat{\alpha}^2} - \hat{\alpha}_h \right]
\]

where, \( \hat{\zeta} = 2(\alpha(\hat{\lambda} - \hat{\lambda})) + (1 - r)K_h(\zeta - 2\hat{\alpha}_h) \).

Define, \( \Delta = \frac{1}{1 - r} \frac{K_h^2}{\hat{\lambda}^2} - \hat{\alpha}_h \). Then, we have \( \frac{\Delta}{\Delta_\alpha} > 0 \), and \( \Delta_\alpha_{\hat{\lambda} \rightarrow 0} < 0 \). Therefore, \( \Delta_\alpha = 0 \) for \( r = r_f \) as defined in the proposition, and strictly negative for all \( r > r_f \). □

PROOF OF COROLLARY 2: The result is proved by noting that the partial derivative of \( r_f \) (defined in (50)) with \( U \) is less than zero. The second part of the Corollary follows from the fact that \( r_f \) decreases as \( K_l - K_h \) difference is higher. □

Appendix B: Pooling Contracts

We prove that pooling contracts will always be dominated for both wholesale price only and fixed-price contracts.

Wholesale Price Pooling Contract

In contrast to wholesale price menu contract, a pooling contract specifies a single wholesale price \( \omega_p \) and quality \( \theta_p \) pair offered to both retailer types. In a pooling contract, the manufacturer chooses the optimal \( (\omega^*_p, \theta^*_p) \) pair that maximize her expected profits while ensuring that both retailer types will have incentives to accept the contract. We note that individual contracts derived in our paper are not pooling contracts as they are designed for one retailer type and as such they do not consider the individual rationality constraint of the other. Under the wholesale price only contracts, the manufacturer’s problem for pooling is:

\[
\Pi_{M}^{\omega} = \max_{(\omega_p, \theta_p)} \left\{ (rK_l + (1 - r)K_h)(\omega_p - c)(\alpha + \lambda \hat{\beta}_p - \hat{\alpha}_p) \right\} - \frac{1}{2} \hat{\theta}^2(\Omega_p)^2.
\]

s.t. \( \frac{1}{2} K_h(\alpha + \lambda \hat{\beta}_p - \hat{\alpha}_p)^2 \geq U \)

\( \frac{1}{2} K_h(\alpha + \lambda \hat{\beta}_p - \hat{\alpha}_p)^2 \geq U \).

The pooling contract is a single contract offered for both types of retailers and hence the incentive compatibility constraints are not needed. We know from the proof of Proposition 1 that under optimal wholesale price contracts, the incentive compatibility constraints (A3 and A4) are binding. Clearly, these constraints are also valid for the above model if they are binding. Hence, the optimal solution to the pooling problem is a feasible solution to the wholesale menu contract problem (13), that is, \( \hat{\alpha}_h = \hat{\alpha}_h = \omega^*_p \) and \( \hat{\theta}_p = \hat{\theta}_p = \hat{\theta}_p \) is feasible solution to (13). Consequently, the optimal manufacturer profit under wholesale pooling contract cannot exceed her profits with wholesale menu contracts and therefore, the pooling contract is dominated. Therefore, from Proposition 2, we conclude that for the case of wholesale price contracts, employing menu of contracts is the best strategy for the manufacturer.

Fixed-fee Pooling Contract

In the case of fixed-fee contracts, pooling is done by offering the fixed fee, \( F_f \), and quality \( \theta_f \). Thus, we define \( B_f = F_f - \xi \theta_f / 2 \). The resulting expected profit function for the manufacturer can be written as follows:

\[
\Pi_{M}^{\theta_f} = \max_{(\xi, B_f)} \left\{ B_f \right\} = \frac{1}{2} \frac{K_h(\alpha + \lambda \hat{\beta}_p - \hat{\alpha}_p)^2}{\xi} - \frac{\theta_f^2}{2} \geq U.
\]

s.t. \( \frac{1}{2} K_h(\alpha + \lambda \hat{\beta}_p - \hat{\alpha}_p)^2 - B_f - \xi \theta_f / 2 \geq U \).

It is easy to show that the second constraint must be binding at optimality since \( K_h < K_l \). Hence, solving for \( \theta_f \), we get \( \theta_f^* = \frac{1}{2} K_h(\alpha + \lambda \hat{\beta}_p - \hat{\alpha}_p) \), and thus, \( \Pi_{M}^{\theta_f} \geq \frac{1}{2} K_h(\xi(\alpha + \lambda \hat{\beta}_p - \hat{\alpha}_p)^2 - U \).

Next, we compare \( \Pi_{M}^{\theta_f} \) with the manufacturer’s profit for the cases of fixed-price individual and menu contracts. First suppose that \( r < K_h / K_l \) in which case the manufacturer’s expected profit under menu contracts, \( \Pi_{M}^{\hat{f}} \) is given in section 3.1.2. We note in section 3.1.3 that \( \hat{\Pi}_{M}^{\hat{f}} \) is increasing in \( r \) and at \( r = 0 \), \( \hat{\Pi}_{M}^{\hat{f}}(r = 0) = \frac{1}{2} K_h(\alpha + \lambda \hat{\beta}_p - \hat{\alpha}_p) - U \geq \hat{\Pi}_{M}^{\hat{f}} \), implying that \( \Pi_{M}^{\theta_f} \geq \frac{1}{2} K_h(\xi(\alpha + \lambda \hat{\beta}_p - \hat{\alpha}_p)^2 - U \).

We know from Proposition 6 that the low-cost individual fixed-price contract dominates the menu of contracts for \( r \geq r_f \) when \( 0 < r_f < K_h / K_l \). The manufacturer’s expected profit under this contract is given by \( \Pi_{M}^{\hat{f}} \). At \( r = r_f \), \( \Pi_{M}^{\hat{f}}(r = r_f) = \frac{1}{2} K_h(\alpha + \lambda \hat{\beta}_p - \hat{\alpha}_p) \), and since \( \hat{\Pi}_{M}^{\hat{f}} \) increases in \( r \), it implies that \( \Pi_{M}^{\theta_f} \geq \Pi_{M}^{\hat{f}}(r_f) \).

Consequently, we conclude that the pooling fixed-price contract will always be dominated by either the fixed-price menu of contracts or the low-cost individual contract.
Franchise Pooling Contract

The proof is analogous to the wholesale price contract case. In the case of franchise-fee contracts, pooling is done by offering the fixed fee \( F_p \), the wholesale price \( \theta_p \), and quality \( \theta_p \). The manufacturer’s optimization model for the pooling contract is

\[
\Pi^*_{M} = \max \left( \alpha \theta_p + \beta \theta_p^2 + \gamma \right) \text{ s.t. } \frac{1}{2} K_i (\alpha + \lambda \theta_p^2 - \tilde{\theta}_p^2) - \tilde{F}_p \geq \theta F_p - \tilde{F}_p.
\]

It is easy to show that the second constraint must be binding at optimality since \( K_2 < K_1 \). We know from Proposition 7 that under optimal franchise menu contracts, the incentive compatibility constraints (A3 and A4) are binding. Clearly, these constraints are also valid for the above model if they are binding. Hence, the optimal solution to the pooling problem is a feasible solution to the wholesale menu contract problem, that is, \( \left( \alpha \tilde{\theta}_p, \beta \tilde{\theta}_p^2, \tilde{F}_p \right) = \left( \alpha \tilde{\theta}_p, \beta \tilde{\theta}_p^2, \tilde{F}_p \right) \) is feasible solution to (50). Consequently, the optimal manufacturer profit under franchise pooling contract cannot exceed her profits with franchise menu contracts and therefore, the pooling contract is dominated. Therefore, we conclude that for the case of franchise contracts, employing menu of contracts is the best strategy for the manufacturer.

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