Multi-stage onboard inventory management policies for food and beverage items in cruise liner operations

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Abstract

We examine optimal policies for multi-stage replenishment of an onboard food and beverage (F&B) item for a cruise liner. Typically, in cruise liner operations, F&B items are ordered from suppliers before the final number of bookings is realized so as to take advantage of price discounts due to advanced contracting. Later, based on the realized headcount, F&B consumption distribution is updated, and subsequently, additional purchases can be made from local spot markets at the origin just prior to the departure and/or at an intermediate stop during the voyage. We investigate and identify optimal contracting and inventory replenishment policies that incorporate contracted and expedited purchasing schemes. We show that while optimal stock market replenishments follow base stock policies, optimal contracting decision can be derived from a piecewise cost function. We discuss insights from our results and propose future research opportunities for similar settings with multiple replenishment instants.

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1. Introduction

In this paper, we consider management of inventory for perishable food and beverage items in the cruise industry where the supply of food is an essential part of the services offered to customers. The motivation of this study originates from our involvement with a major cruise line headquartered in the South Eastern part of the United States. The cruise line under study manages its procurement process by establishing contracts with distributors/vendors in the designated home port and/or purchasing products through expedited replenishment orders from spot markets. Minimizing the likelihood of stock-outs, being protected against fluctuations in the market, and price discounts are basic incentives for cruise lines to sign contracts with various distributors. However, such contracts need to be signed in advance to assure timely delivery of the products with lower cost.

Oftentimes, the supply contract for an F&B product is signed before an accurate number of customers (bookings) for the cruise is known. As the cruise ship nears its departure time, the demand information becomes more accurate. If there is a need to increase inventory, additional procurements are made through expedited orders from local spot markets. It is our observation that expedited replenishments from spot markets, although more costly, play a critical role in the provisioning of food and beverage items in the cruise line industry. While advance supply contracting provides cost efficient procurement opportunities, spot market purchases serve as a hedge against stock-outs due to variations across day-to-day demands. Therefore, an efficient inventory control
mechanism must take in consideration the trade-offs between both replenishment approaches.

In this study, we propose a stochastic dynamic programming model to optimize onboard inventories of food and beverage items for a cruise liner. The proposed model incorporates multiple replenishment stages comprised of both contracted and expedited purchases. In service sectors such as the cruise line industry, the availability of products is an essential part of the quality of services provided to the customers. Therefore, purchasing and inventory decisions play a central role in planning the operations. In a recent survey, Stanley and Wisner (2001) established a strong relationship between purchasing operations and service quality to external customers. Researchers such as Iyer and Bergen (1997), Kouvelis and Gutierrez (1997) and Gurnani and Tang (1999) consider multiple replenishment instants as part of a newsvendor setting where the purchases are carried out in two stages to utilize price discounts and forecast updates similar to our case. However, our model differs from these papers in two aspects. First, in the aforementioned work, the objective is to maximize profits, while we focus on cost minimization. Newsvendor settings primarily focus on direct sales and thus the main motivation is to increase revenues from sales. On the other hand, on a cruise liner the main goal of carrying food inventory is to achieve higher customer satisfaction through availability of products while being cost efficient. Second, in those models, all sales take place after purchases from suppliers are finalized (single period setting) whereas we consider option of replenishment while the good is already being consumed (multi-period setting).

In parallel to the increasing trend in operations research literature towards analyzing service sector related problems, food logistics emerges as one of the most appealing research topics in this area in recent years. The focus spans production planning and logistics in food processing industries (van Donk, 2001), efficient distribution of foods (Lijima, Komatsu, & Katoh, 1996; Tarantilis & Kiranoudis, 2001) and food supply chains (Henson, Loader, & Traill, 1995; van der Vorst, Beulens, & van Beek, 2000). The work by Evangelos, Hill, Saraf, and Miller (1998) is the only one that we are aware of focusing on food logistics under the context of maritime operations. In their paper, authors present a software tool that “optimizes” Navy menus based on food cost, labor hours and storage requirements. We believe that our research contributes to the related literature by opening up a new venue that deals with multi-stage inventory management of perishables in food logistics and maritime operations.

The rest of the paper is organized as follows: in Section 2 we present the description of our model. Optimal replenishment policies are derived in Section 3. Section 4 presents numerical examples. We provide insights and discuss various extensions to the model in Section 5 and conclude the paper in Section 6.

### Nomenclature

\[
\begin{align*}
  y_j & \quad \text{purchase amount at stage } j; j = 0, 1, 2 \\
  c_j & \quad \text{unit purchase price of the F&B item at stage } j \\
  p & \quad \text{per-unit stock out cost} \\
  h & \quad \text{per-unit leftover cost at the end of the cruise trip} \\
  t_i & \quad \text{time elapsed during the } i\text{th leg of the cruise trip; } i = 1, 2 \\
  B & \quad \text{number of bookings (home port demand) for the cruise trip} \\
  s & \quad \text{scenario index for the home port demand} \quad (s = L, H) \\
  q_s & \quad \text{probability for scenario } s \\
  B_s & \quad \text{number of bookings under scenario } s \\
  D_t & \quad \text{onboard demand for the F&B item during time } t \\
  f(D/B) & \quad \text{probabilistic density function for demand over } t \text{ given } B \\
  F(D/B) & \quad \text{cumulative distribution function for demand over } t \text{ given } B \\
  I & \quad \text{excess onboard inventory at the beginning of stage 2} \\
  S_j(x) & \quad \text{expected total cost of leftovers and stock outs given inventory level, } x, \text{ at the beginning of stage } j \\
  G_j & \quad \text{expected total cost at stage } j \\
  K_j & \quad \text{order-up-to level for stage } j
\end{align*}
\]
bottage logistics is on the former two parameters. In particular, we consider a multi-stage model that focuses on maintenance of day-to-day availability of a F&B item during a cruise trip in a cost-efficient way.

The issue of availability in this context has distinguishing characteristics in several aspects. First, a considerable portion of the food services provided to the ship's guests is part of the vacation package and thus the demand originates from a finite set of customers. This differentiates the ship's inventory problem from that of a traditional grocery store or supermarket. Second, the cruise line must have fresh products to offer its guests throughout the entire cruise. Moreover, a guest on a ship is quite different than a customer who visits a convenience store. Guests pay a fee for the entire vacation package and thus dissatisfaction caused by the stock-out of a F&B item does not only lead to lost sales, but also a loss of the customer's goodwill from an extended perspective. As a result, the consequences of stock-outs can be very costly in this environment. Finally, the F&B inventory on a cruise ship can be replenished only at the port of origin or at an intermediary stop. The time and cost associated with this replenishment may be very extensive and varies based on the source.

The F&B item studied in this paper is perishable and is disposed of if not consumed at the end of the voyage. Therefore, excess stocks cannot be transferred to another ship's inventory. The cruise line incurs a disposal cost associated with the unconsumed item. We assume that the item stays fresh and consumable during the voyage. We study a cost minimization problem with three-stage procurement decision process. In stage 0, after the cruise is scheduled and before the final number of bookings is realized, an initial purchase amount, \( y_0 \), is contracted under unit price of \( c_0 \). Similar to the procurement of many goods, contracting the purchases with suppliers in advance incurs cost savings due to price deductions in this environment. At stage 1, just prior to the departure date, final bookings are realized and the contracted amount of the F&B item is delivered. Based on the updated information regarding the number of guests to be boarded, the cruise liner might need to adjust its inventory via additional purchase orders from local suppliers (spot market) in the home port city. However, a higher unit price, \( c_1(c_1 > c_0) \) needs to be paid. The cruise liner departs the origin port and continues its route until it stops at an intermediate location where another local (spot) market exists for additional replenishment of the F&B item if needed. The replenishment choice at this phase constitutes the stage 2 decision.

The trip to the intermediate stop lasts \( t_1 \) days during which a total demand of \( D^1 \) for the item is realized. However, the F&B managers avoid this option as much as possible since the per unit price, \( c_2 \), is typically prohibitive in these markets as they are situated in relatively remote locations. In the rest of the paper we assume that \( p > c_2 > c_1 > c_0 \) which is quite a realistic assumption in this environment. Finally, the cruise liner needs to travel for \( t_2 \) days to end its journey back at home port. Thus, the demand for this leg of the voyage is \( D^2 \). The procurement amounts in stages 2 and 3 are denoted by \( y_1 \) and \( y_2 \) respectively. The sequence of events on a timeline is exhibited in Fig. 1. As we will demonstrate in Section 5 while the analysis can be extended to more than three stages with little modification, our qualitative results will remain unchanged.

In stage 0, the company planners face two sources of uncertainty: (1) the number of guests, and (2) the total amount of consumption during the cruise trip. As pointed out earlier, the company may be better off contracting the purchase of F&B items in advance, before the final number of bookings is realized, so as to take advantage of price deductions. We model the probabilistic outcome of the number of bookings, \( B \), by a discrete distribution. We consider two possible outcomes, namely, \( B_H \) and \( B_L \), where \( B_H > B_L \), with probabilities \( q_H \) and \( q_L(q_H + q_L = 1) \) respectively. Modeling stochastic demand using discrete scenarios is a common

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**Fig. 1.** Timeline and stages of the replenishments for the F&B item.
approach in literature (see for example Huchzermeier & Cohen, 1996; Agrawal, Smith, & Tsay, 2002). We employ the two-scenario modeling approach to facilitate our analysis and for sake of brevity. Clearly, since the home-port demand for the cruise ship is always an integer, by expanding the scenarios we can account for more potential demand values. As we will explain in Section 5, extending our analysis to multiple demand scenarios can be achieved with little modification and will lead to no significant changes in main qualitative results obtained by the two-scenario model. Our model assumes that the total number of guests is realized at the end of the initial stage. However, daily consumption remains to be uncertain. In our setting, total demand over t days, $D_t$, for a given number of guests, follows a non-negative continuous probabilistic density function $f(D_t | B_t)$ with a cumulative distribution function (cdf) denoted by $F_{t}(D_t | B_t)$ implying stochastic dominance. We also assume that demand during the cruise vacation is independently and identically distributed (i.i.d.) across days. Therefore, distribution for demand over t days is simply the convolution of t i.i.d. distributions. Moreover, we let $F (D | B)$ denote the tail distribution of demand over t days (i.e., $F (D | B) = 1 - F(D | B)$).

At each stage, the cruise liner must decide on the replenishment amount for the F&B item. The decision at each stage hinges on the inventory level at hand at the beginning of the stage and the prospect on the future demand in addition to various cost factors. Besides procurement costs, we consider a per-unit stock-out cost, $p$ and leftover cost, $h$. A stock-out cost occurs when demand exceeds available inventory. In most part, $p$ captures the loss of goodwill of the customers due to shortages in F&B services. It is assumed that it stays constant during the voyage. The latter cost is due to having excess inventory at the end of the voyage. Clearly, the leftovers need to be disposed of as they cannot be kept fresh until the next trip because of perishability. As a result the cruise company incurs cost for discarding the excess inventory. In addition, $h$ might include opportunity cost due to overstocking. The day-to-day holding costs are ignored in our analysis as they are generally negligible compared to other cost items and the total voyage time spans mostly a period of few days to two weeks.

3. Optimal replenishment policies

To determine the optimal replenishment policies we utilize the probabilistic dynamic programming paradigm with backward induction. In this approach we first investigate optimal stage 2 orders given the inventory level at time of docking of the ship at the intermediate port. We then derive optimal policies for stage 1 and stage 0 procurements. In the rest of the analysis, $E[x]$ is used for expected value of $x$ and $x^*$ is equivalent to max(0, $x$).

3.1. Stage 2 problem: replenishment at the intermediate stop

At this stage, the replenishment decision hinges on current inventory level which is denoted by $I$. Clearly, $I$ is a function of $y_0$, $y_1$ and $D_t^1$. At the end of the first leg of the voyage, $D_t^1$ is realized and stock level drops to $I = (y_0 + y_1 - D_t^1)^+$. Obviously, a new replenishment amount, $y_2$, in this stage will bring the inventory level to $I + y_2$. Thus, we can compute the expected sum of the leftover cost at the end of the voyage and the stock out cost at the final leg of the voyage, $S_2$, as follows:

$$S_2(y_2 + I) = hE[(y_2 + I - D_t^2)^+] + pE[(D_t^2 - y_2 - I)^+]$$

$$= h \int_0^{y_2+I} (y_2 + I - x)f_{t}^{2}(x | B)dx + p \int_{y_2+1}^{\infty} (x - y_2 - I)f_{t}^{2}(x | B)dx$$

$$= h \int_0^{y_2+I} F_{t}^{2}(x | B)dx + p \int_{y_2+1}^{\infty} F_{t}^{2}(x | B)dx.$$ 

Next we can write down the total expected cost function for stage 2, which we denote by $G_2$

$$G_2(y_2, I) = c_2y_2 + S_2(y_2 + I).$$

The objective in this stage is to compute the optimal $y_2$ that minimizes the foregoing cost function, given $I$. In order to justify using first order optimality conditions, we need to show first that $G_2$ is convex in $y_2$

$$\frac{\partial^2 G_2}{\partial y_2^2} = (h + p)f_{t}^{2}(y_2 + I | B).$$

It is straightforward to see that the foregoing second order derivative always returns a non-negative value indicating that the third stage cost function is convex in $y_2$. Thus, we can derive the optimal replenishment amount by solving the following first-order optimality condition:

$$\frac{\partial G_2}{\partial y_2} = c_2 + (h + p)f_{t}^{2}(y_2^* + I | B) - p = 0,$$

which leads to

$$F_{t}^{2}(y_2^* + I | B) = \frac{p - c_2}{p + h}.$$ 

Eq. (3) can be regarded as the so-called critical fractile for the stage 2 problem. To further specify the optimal ordering policy, let $K_2$ be such that

$$K_2 = F_{t}^{2-1}(\frac{p - c_2}{p + h} | B).$$
Note that \( F^{(i)}(x \mid B) \) is an increasing function of \( x \), and thus, if \( I \) is large enough, specifically greater than \( K_2 \), then \( G_2 \) increases in \( y_2 \) everywhere, implying that no purchase should be made. Consequently, we can give the optimal order amount as follows:

\[
y^*_2 = \begin{cases} 
0 & I \geq K_2 \\
K_2 - I & \text{otherwise.}
\end{cases} \tag{5}
\]

The result in (5) suggests that the cruise liner is better off ordering at stage 2 if and only if the current inventory level is below a certain threshold (i.e., \( K_2 \)), which is often-times referred to as the base stock level. The threshold value is independent of the order amounts of the previous stages, however, it increases in \( p \) and decreases in \( c_2 \) and \( h \).

### 3.2. Stage 1 problem: expedited replenishments in home market

At this stage the cruise liner is about to depart from the home port and the number of bookings, \( B \) is realized. The goal in this stage is to determine the amount of orders from the local spot market \( y_1 \) that minimizes the total expected cost over stages 1 and 2, given \( y_0 \). Similar to the stage 2 analysis, we first compute the total expected leftover and stock-out costs, say \( S_j \), for stages 1 and 2. We, then, write down the total cost function, \( G_j \).

\[
S_1(y_0 + y_1) = pE[(D_{y_1} - y_0 - y_1)^+] + S_2(E[\max(K_2, y_0 + y_1 - D_{y_1})]). \tag{6}
\]

\[
G_1(y_0, y_1) = c_1y_1 + S_1(y_0 + y_1) + c_2E[\max(0, K_2 - (y_0 + y_1 - D_{y_1})^+)]. \tag{7}
\]

Our first conclusion regarding the optimal stage 1 decision, \( y^*_1 \), is captured by the following lemma:

**Lemma 1.** Assuming \( c_2 > c_1 \), then \( y^*_1 \geq K_2 - y_0 \).

**Proof.** Suppose \( y_1 < K_2 - y_0 \). Then we can rewrite (6) and (7) as follows:

\[
S_1(y_0 + y_1) = pE[(D_{y_1} - y_0 - y_1)^+] + S_2(K_2),
\]

\[
G_1(y_0, y_1) = c_1y_1 + S_1(y_0 + y_1) + c_2 \int_{0}^{y_0+y_1} (K_2 - (y_0 + y_1 - x)) f^{(i)}(x \mid B) dx + c_2 K_2 F^{(i)}(y_0 + y_1 \mid B).
\]

To see the marginal impact of changes in \( y_1 \) on \( G_1 \), we can look at the first derivative:

\[
\frac{\partial G_1}{\partial y_1} = (p - c_2) F^{(i)}(y_0 + y_1 \mid B) - (p - c_1). \tag{8}
\]

Clearly the derivative is negative for any non-negative \( y_1 \) since \( c_1 < c_2 \). Thus, the total costs at stage 1 decrease in \( y_1 \) implying that optimal replenishment amount cannot reside below \( K_2 - y_0 \). □

The foregoing observation points out that the total orders in stages 0 and 1 will exceed the critical quantile in stage 2 at optimality. This result is consistent with the real life practice where logistics managers in various cruise lines prefer to cover the demand via stage 0 and 1 purchases, due to high prices at midway markets. The result is valid regardless of the relationship between \( t_1 \) and \( t_2 \). Based on the results of Lemma 1, one should look for optimal above \( K_2 - y_0 \) in which case (6) and (7) can be expanded as follows:

\[
S_1(y_0 + y_1) = p \int_{y_0+y_1}^{\infty} F^{(i)}(x) dx + \int_{0}^{y_0+y_1-K_2} S_2(y_0 + y_1 - x) f^{(i)}(x \mid B) dx + S_2(K_2) F^{(i)}(y_0 + y_1 - K_2 \mid B). \tag{9}
\]

\[
G_1(y_0, y_1 \mid y_1 \geq K_2 - y_0) = c_1y_1 + S_1(y_0 + y_1) + c_2 \int_{y_0+y_1-K_2}^{y_0+y_1} (x - (y_0 + y_1 - K_2)) f^{(i)}(x \mid B) dx + c_2 K_2 F^{(i)}(y_0 + y_1 \mid B). \tag{10}
\]

Next we compute the first derivative of \( G_1 \):

\[
\frac{\partial G_1}{\partial y_1} = c_1 - p F^{(i)}(y_0 + y_1 \mid B) + \int_{0}^{y_0+y_1-K_2} S_2'(y_0 + y_1 - x) f^{(i)}(x \mid B) dx - c_2 [F^{(i)}(y_0 + y_1 \mid B) - F^{(i)}(y_0 + y_1 - K_2 \mid B)], \tag{11}
\]

where

\[
S_2'(y_0 + y_1 - x) = (p + h) F^{(i)}(y_0 + y_1 - x \mid B) - p. \tag{12}
\]

Using (11) we can show that the foregoing equation is strictly convex in \( y_1 \), implying that there is a unique minimizing stationary point in \( G_1 \).

**Lemma 2.** \( G_1 \), given in (10), is strictly convex in \( y_1 \).

**Proof.** See the Appendix A. □

A straightforward analysis of (10) and (11) along with the foregoing lemma reveals that there is a unique value for \( y_0 + y_1 \) for which the stage 1 expected cost function is minimized. Let \( K_1 \), denote this value. Clearly for \( y_0 + y_1 = K_1 \), the following first-order optimality condition must be satisfied

\[
(p - c_2)F^{(i)}(K_1 \mid B) + (p + h) \times \int_{0}^{K_1-K_2} f^{(i)}(K_1 - x \mid B) F^{(i)}(x \mid B) dx - (p - c_1) = 0 \tag{13}
\]

Notice that the foregoing function is derived from (11) and (12), and integration by parts. Although, it may
not be possible to derive a closed form definition for $K_1$, the numerical value can be found via a simple search since $G_1$ is proved to be convex. From Lemmas 1 and 2, we deduce that $K_1 > K_2$. This leads us to the following replenishment policy that is optimal for stage 1

$$y_1^* = \begin{cases} K_1 - y_0 & \text{if } y_0 < K_1 \\ 0 & \text{otherwise} \end{cases}$$

(14)

Thus, we conclude that $K_1$ is the stage 1 optimal base stock level for the F&B item. From Eq. (13), we note that the stage 1 base stock level increases in $p$, $c_2$, $t_1$ and $t_2$ and decreases in $c_1$ and $h$.

3.3. Stage 0 problem: advance contracting to supply

In addition to daily demand, the finalized bookings, $B$, are also uncertain at this stage. The amount contracted to the supplier, $y_0$, must be chosen before $B$ becomes certain. Two possible outcomes are $B_H$ and $B_L$ with probabilities $q_L$ and $q_H$ ($q_L + q_H = 1$) respectively. The indices $H$ and $L$ are used to characterize high and low demand realizations. First thing to notice is that both $K_1$ and $K_2$ depend on the realized value of $B$ since the onboard demand distribution of the F&B item is a function of $B$.

Therefore, in the rest of the paper, we will denote base stock levels for stages 1 and 2 with $K_i(B_j)$ where $i = 1, 2$ and $j = H, L$. The relation between stage 1 and 2 base stock levels and $B$ is established by the following Lemma.

Lemma 3. Assuming stochastic ordering for $F(X)$, $K_i(B_H) > K_i(B_L)$ for $i = 2, 3$.

Proof. See the Appendix A.

The aforementioned result simply suggests that the base stock levels must be higher at both downstream stages when the number of guests on ship is higher. The initial stage is different from the other two since this stage's order is not delivered to the cruise liner until the beginning of the next decision stage, and meanwhile, no demand for the F&B item occurs. With this given, the analysis of stage 1 indicates that the company should not order in stage 1 if $y_0$ exceeds the stage 1 base stock level. However the “right” base stock levels for next stages are not known yet at stage 0 in definite terms. It should be underlined that additional purchases at stage 1 are solely due to the fact that $B$ is realized only after stage 0. Obviously, when $B$ is deterministic, no purchase will be needed at stage 1 since $c_1 > c_0$. Optimal order amount at stage 0 for the deterministic case, $K_0(B)$, would be still computed via (13), except that $c_1$ would be replaced by $c_0$. Obviously, $K_0(B) > K_j(B)$ for any $j$. On the other hand, when $B$ is an outcome of multiple possible scenarios with discrete probabilities, the expected cost function for the initial stage will then be

$$G_0(y_0) = c_0 y_0 + \sum_{j=H,L} q_j \left( S_1(y_0, K_1(B_j)) + c_1(K_1(B_j) - y_0)^+ \right) + c_2 E \left[ (K_2(B_j) - (y_0, K_1(B_j)) - D^j(B_j))^+ \right]^+$$

(15)

The following theorem provides a general framework for optimal contracting amount at the initial stage.

Theorem 1. Assuming $q_L, q_H \in (0, 1)$ and $c_0 < c_1$, $G_0(y_0)$ is unimodal and the unique stationary point corresponds to the global optimum, $y_0^*$, which resides in $(K_0(B_L), K_0(B_H))$.

Proof. First, notice that $K_0(B_L) < K_0(B_H)$ is from stochastic ordering assumption. Observe that the expected cost function decreases in $[0, K_0(B_H)]$ since $K_0(B_H)$ is the optimal order amount when $B$ is deterministic and known to be $B_L$. However, in the stochastic case, it is the worse case scenario and thus the expected number of guests to be covered is larger. Therefore, due to stochastic dominance and convexity, the expected costs must be decreasing for any value less than or equal to $K_0(B_L)$. A similar rationale can be used to show that $G_0$ is increasing in $(K_0(B_H), \infty)$.

Next, suppose $K_0(B_L) \geq K_1(B_H)$. Then, $y_0 \leq K_1(B_H)$ indicating that no purchases will be necessary at stage 1. Consequently, $G_0$ can be written as follows:

$$G_0(y_0) = c_0 y_0 + \sum_{j=H,L} q_j \left( S_1(y_0) + c_2 E \left[ (K_2(B_j) - (y_0, D^j(B_j))^+ \right]^+ \right).$$

(16)

Observe that the second derivative with respect to $y_0$ will then be

$$\frac{\partial^2 G_0}{\partial y_0^2} = \sum_{j=H,L} q_j \left( (p - c_2) f^{(2)}(y_0 \mid B_j) + (p + h) \times \int_0^{y_0 - K_2(B_j)} f^{(2)}(y - x \mid B_j) f^{(1)}(x \mid B_j) dx \right).$$

Obviously, the foregoing function returns a positive value implying that the expected cost function is convex in $(K_0(B_L), K_0(B_H))$ and there must be a unique stationary point in this region. Now suppose $K_0(B_L) < K_1(B_H)$. Then the expected cost function is piecewise in this region. We can easily deduce from the above analysis that the cost function is convex in $(K_1(B_H), K_0(B_H))$. For $(K_0(B_L), K_1(B_H))$, the expected cost function is

$$G_0(y_0) = c_0 y_0 + q_L \left( S_1(y_0) + c_2 E \left[ (K_2(B_L) - (y_0, D^j(B_L))^+ \right]^+ \right) + (1 - q_L) \left( S_1(K_1(B_H)) + c_1(K_1(B_H) - y_0)^+ \right) + c_2 E \left[ (K_2(B_L) - (K_1(B_H) - D^j(B_H))^+ \right]^+ \right),$$

(17)
and the second derivative is
\[ \frac{\partial^2 G_0}{\partial y_0^2} = -q_L \left[ (p - c_2) f^{(1)}(y_0 \mid B_L) + (p + h) \right. \]
\[ \times \left. \int_{y_0}^{\infty} f^{(1)}(y_0 - x \mid B_L) f^{(1)}(x \mid B_L) \, dx \right]. \]

which is also positive. One can easily see that in both cases the marginal cost at \( K_1(B_H) \) are similar and hence, there can be one and only one stationary point for \( G_0 \) for any \( y_0 \geq 0 \). \( \square \)

In summary, the foregoing theorem implies that there exists a unique optimal contract amount, \( y_0^* \), which resides in \((K_0(B_L), K_0(B_H))\). Unfortunately, we cannot derive a closed form equation for \( y_0^* \). Let \( \hat{K}_0(y_0 \mid B_j) \) be a function of \( y_0 \) such that:
\[ \hat{K}_0(y_0 \mid B_j) = F^{(1)}(y_0 \mid B_j) + \frac{1}{F^{(2)}(K_2(B_j) \mid B_j)} \]
\[ \times \int_{y_0}^{\infty} f^{(2)}(y_0 - x \mid B_j) f^{(1)}(x \mid B_j) \, dx. \]

Then, from (3), (13), (16) and (17), \( y_0^* \) must satisfy one and only one of the following two equations:
\[ q_L \left[ (p - c_2) \hat{K}_0(y_0^* \mid B_L) - (p - c_1) \right] - (c_1 - c_0) = 0, \]
\[ (p - c_2) \hat{K}_0(y_0^* \mid B_L) + (1 - q_L) \hat{K}_0(y_0^* \mid B_H) - (p - c_0) = 0. \]

Specifically, if
\[ \hat{K}_0(K_1(B_H) \mid B_L) > \frac{q_L(p - c_1) + (c_1 - c_0)}{q_L(p - c_2)}, \]
then (18) must hold at optimality. Otherwise, (19) holds. In the former case, \( y_0^* \leq K_1(B_H) \) and for the latter case, \( y_0^* \geq K_1(B_H) \). Both equations hold if and only if \( y_0^* = K_1(B_H) \). It is straightforward to see that the right-hand side in (20) decreases in both \( q_L \) and \( c_0 \) implying that for sufficiently small values of \( q_L \) and \( c_0 \), stage 0 orders will exceed \( K_1(B_H) \). Moreover, using chain rule in partial derivation, one can show that the right-hand side in (20) is decreasing while the right-hand side is increasing in \( c_1 \) and \( c_2 \). Therefore, we can conclude that it is likely that \( y_0^* > K_1(B_H) \) for relatively higher \( c_1 \) and \( c_2 \). In those cases, no replenishment is needed in stage 1 regardless of the realized value of \( B \). Clearly, stage 2 replenishments depend on the realized demand at the end of the first segment of the voyage.

4. Numerical example

To illustrate the relationship among the contract amount, contract price, and home port demand distribution, we provide a numerical example where the daily demand for an F&B item by one guest follows an exponential distribution with a mean of 1 unit. The parameters of the example are given below:
\[ p = 9 \quad c_1 = 1.5 \quad c_2 = 2 \quad h = 0.25, \]
\[ t_1 = 3 \quad t_2 = 4 \quad B_L = 10 \quad B_H = 20. \]

Notice that since demand distribution is i.i.d., with \( B \) customers, demand over \( t \) days follows, \( B t \)-Erlang(1). Consequently, base stock levels at stages 1 and 2 are as follows:
\[ K_1(B_L) = 73 \quad K_1(B_H) = 145, \]
\[ K_2(B_L) = 44 \quad K_2(B_H) = 86. \]

We compute initial contract amount under three different prices, \( c_0 = 0.2, 0.5 \) and 1, while varying \( q_L \) in [0,1]. The results are depicted in Fig. 2. Observe that in all cases, the contract amount is above \( K_1(B_H) \) for small values of \( q_L \) implying that no replenishment will be necessary at stage 1 when the probability for low demand realization is low. As the probability increases the contract amount decreases. The decrement becomes exponential for all cases after a certain point. Clearly, if the cost of contracting is relatively high, the purchase amount decreases below \( K_1(B_H) \) line at a relatively small \( q_L \) value. In this case, some of the purchases are delayed to stage 1 as the risk of buying in advance is high due to high stage 0 cost. Still, the contract amount is always beyond \( K_1(B_L) \) indicating that additional purchase takes place in stage 1 only if the realized number of bookings is \( B_H \). For small values of \( c_0 \), the contract amount stays above \( K_1(B_H) \) even when \( q_L \) is as low as 0.5. This suggests that the portion of purchases delayed until stage 1 increases in \( c_0 \).

We also observe that the impact of contracting cost on the contract amount decreases as \( q_L \) approaches either to zero or one. The change in contract amounts under varying initial stage costs is relatively small in
those cases. However, opposite is observed when \( q_L \) is closer to 0.5. Moreover, both our analytical and numerical results indicate that the impact of contract price is higher when the gap between \( B_L \) and \( B_H \) increases. Obviously both our analytical and numerical results indicate that the impact of contract price is higher when the gap between \( B_L \) and \( B_H \) increases. Consequently, these results hint that the procurement decisions of the purchasing managers will be more sensitive to the contract price when the demand for the cruise is volatile (or when they are less informed regarding the market potential). Such insight will be especially valuable for 3PL providers and/or so called “ship chandlers”, who provide F&B products to cruise ships, when designing their supply contracts.

5. Additional discussion: insights and extensions

The analysis in the previous section indicates that whether expedited replenishments at the origin and the intermediate locations are needed or not is context specific. Depending on the relationships among various cost factors, the optimal policy can lead to one of several possible scenarios depicted in Fig. 3.

Results of stage 1 analysis indicates that for a realized value of \( B \), if the stage 1 base stock level, \( K_1(B) \), is below the initial contract amount, \( y_0 \), then no replenishment should be made at stage 1 (Fig. 3a and b). Such outcome is likely to arise if the gap between contract price at stage 0 and cost of expedited replenishments at downstream stages are large (i.e., \( c_0 \ll c_1 \) and/or \( c_0 \ll c_2 \)) and when the probability for scenario(s) with higher \( B \) is sufficiently high. Scenarios depicted in Fig. 3c and d is likely to arise in opposite cases.

The amount of excess inventory at the end of stage 1 depends on the realized demand on the F&B item during the first leg of the voyage which takes \( t_1 \) days. Similar to the stage 1 case, optimal policy suggests a base stock level, \( K_3(B) \), for stage 2 as well. If the inventory on hand is beyond this level at the end of stage 1, there is no need for replenishments at stage 2 (Fig. 3a and c). These scenarios are more likely to occur when stage 2 prices are much higher than stage 1 and/or stage 0 prices (i.e., \( c_2 \gg c_1 \) and/or \( c_2 \gg c_0 \)).

As we mentioned earlier, extending the analysis to a more general case with more than three stages is straightforward. To see this, suppose there are \( n(n > 3) \) replenishment stages where \( n - 2 \) of those are intermediate stops. If the procurement cost at stage \( i (n \geq i > 2) \) is higher than the following stage then stage \( i \) problem becomes similar to the last stage (stage 2) problem in our model with the exception that no left-over cost is included in the computation. Consequently, the critical fractile at this stage will be \( (\frac{p_1 - c_i}{p}) \). On the other hand, if the cost is lower at stage \( i \) compared to the subsequent stage then the stage \( i \) can be solved using a similar approach employed in stage 1 of our model. The only exception would be that we substitute \( y_0 \) in stage 1 computations with excess stocks, say \( I_{n+1} \), carried from the previous stage. Although mathematical computations will become more complex, the qualitative results and managerial insights obtained in our model remain intact under the general case.

![Fig. 3. Optimal replenishment scenarios: (a) \( y_1 = y_2 = 0 \); (b) \( y_1 = 0 \) and \( y_2 > 0 \); (c) \( y_1 > 0 \) and \( y_2 = 0 \); (d) \( y_1 > 0 \) and \( y_2 > 0 \).]
5.1. n-Scenario case for B

The results of stage 0 can be generalized to multiple (i.e., more than 2) scenario cases for B. Suppose there are \( n \) possible outcomes for \( B \) with positive discrete probabilities where \( n > 2 \). In this case, the initial stage cost function becomes an \( n + 1 \)-piecewise function. Let all outcomes of \( B \) be ordered in non-increasing fashion, that is, \( B_i < B_{i+1} \) for all \( i = 1, \ldots, n-1 \). A straightforward analysis of Theorem 1 will reveal that the optimal contract amount must be in \( (K_d(B_1), K_d(B_n)) \). Specifically, the results of Theorem 1, unimodularity, (18), and (19) indicate that there exist two outcomes, \( B_m \) and \( B_{m+1} \) where \( 1 \leq m \leq n \) such that \( K_1(B_m) \leq \gamma_0 \leq K_1(B_{m+1}) \). In this interval

\[
G_0(\gamma_0) = c_0\gamma_0 + \sum_{i=1}^{m} q_i \left( S_1(\gamma_0) + c_2E \left[ (K_2(B_i) - (\gamma_0 - D_i^{(B)})^+) \right] \right) \\
+ \sum_{j=m+1}^{n} q_j \left( S_1(K_1(B_j)) + c_1(K_1(B_j) - \gamma_0)^+ c_2 E \right) \\
\times \left[ (K_2(B_j) - (K_1(B_j) - D_i^{(B)})^+) \right]^{+} \right)
\]

and following first-order optimality condition must hold:

\[
G'_0(\gamma_0 | m) = (p - c_2) \sum_{i=1}^{m} q_i \left( \bar{K}_0(\gamma_0 | B_i) \right) - P_m(p - c_1) - (c_1 - c_0) \]

where \( P_m \) is the probability that the number of customers will not exceed \( B_m \). In order to compute \( \gamma_0 \), one needs to identify \( m \) first. Fortunately, due to unimodularity of the stage 0 cost function, we do not need to compute \( K_1 \) and \( K_2 \) values for all \( B_m, j = 1, \ldots, n \) so as to determine the interval where optimal solution resides. It is sufficient to check the value of \( G_0(K_1(B_j) | j) \) for each \( B_j \) starting form \( j = 1 \) until a non-negative outcome is reached. To achieve this we provide a simple search algorithm that can be used when \( n > 2 \):

**Step 1.**
Compute \( K_2(B_1) \) and \( K_1(B_1) \) from (4) and (13).

\( j \leftarrow 1 \); 

**Step 2.**
Do 

\( j \leftarrow j + 1 \)

Compute \( K_2(B_j) \) and \( K_1(B_j) \) from (4) and (13).

\( \) while \( G'_0(K_1(B_j) | j) < 0 \); 

**Step 3.**
Solve (22) for \( \gamma_0 \) given \( m = j - 1 \).

\[\text{Let } K_1(B_{m+1}) = K_d(B_n)\]

5.2. Contracting for multiple ships

On several occasions, especially during high demand season, multiple ships from the same cruise line may be departing from the same port of origin for varying destinations within a short time frame (e.g. couple of days). Clearly, for each cruise line, decisions made in stages 1 and 2 are independent since they are providing services to mutually exclusive markets (i.e., their own customers onboard). However, risk associated to uncertainty regarding number of bookings can be pooled at stage 0 by aggregating all stage 0 orders for the F&B item in the supply contract. Let \( z \) denote the number of cruise liners to be included in the stage 0 supply contract. Consider again \( n \)-scenario bookings, where for cruise liner \( l \), the finalized bookings can turn out to be \( B_l^i \) with a probability of \( q_l^i \) for \( j = 1, \ldots, n \), respectively. Assuming that there is no correlation between cruise line bookings, there will be \( n^z \) possible scenarios. Also let \( s \) represent the scenario index and define:

\[
\Omega_s = \sum_{l=1}^{z} K_1(B_l^i)
\]

for all \( s = 1, \ldots, n^z \), where \( B_l^i \) is the realized bookings for cruise liner \( l \) in scenario \( s \). Note that \( \Omega_s \) is the sum of stage 1 base stock levels over all cruise liners. First, observe that if the initial contract amount turns out to be smaller than \( \Omega_s \), a realized scenario, stage 1 purchases will pull stocks of each cruise liner to its stage 1 base stock level. However, if \( \gamma_0 > \Omega_s \) then the question of how to allocate the excess purchases to ships arises at stage 1. We leave the optimal allocation problem for future study and assume uniform allocation in this analysis for simplicity: each ship receives extra inventory of \( (\gamma_0 - \Omega_s)/z \). Now we can write the stage 0 cost function

\[
G_0(\gamma_0) = c_0\gamma_0 + m \sum_{s=1}^{z} \sum_{l=1}^{n} \left( S_l \left( K_1(B_l^i) + \frac{\gamma_0 - \Omega_s}{z} \right) + c_2 E \left[ (K_2(B_l^i) - (\gamma_0 - D_l^{(B)})^+) \right] \right) \\
+ \sum_{s=1}^{n^z} \left( \rho_s (c_1(\Omega_s - y) - y) \right) \sum_{l=1}^{z} \left( S_l(K_1(B_l)) + c_2 E \left[ (K_2(B_l^i) - (K_1(B_l^i) - D_l^{(B)})^+) \right] \right),
\]

where \( \rho_s \) is the probability for the realization of scenario \( s \) which can be calculated through product of all ships’ booking probabilities. Here, \( t_l^i \) and \( c_2^i \) are the length of the first part of the voyage for cruise liner \( l \) and unit purchase price of the F&B item at the intermediate stop visited by cruise liner \( l \), respectively. It must be underlined that Eq. (23) captures only a certain region of the overall
cost function defined by \( m \) similar to (17) and (21). In (23), it is suggested that the contract amount will exceed the total stage 1 base stock levels in the first \( m \) scenarios. It can be shown and verified from Theorem 1 and (23) that the cost structure in all three cases are similar in the sense that optimal contract amount \( y_0^* \) covers all base stock levels of stage 1 for some scenarios while additional purchases will be needed in others. Therefore, the optimal solution can be found on a piecewise function using a similar approach employed in Sections 3.3 and 5.1.

Due to the fact that the size of the problem increases exponentially with an increase in the number of ships, calculations will be much more cumbersome compared to cases discussed earlier as the number of ships increases. However, in reality, the number of ships departing the same port within a short time period is mostly very small. Also it must be underscored that bookings (i.e., home port demand for cruise liners) across ships are generally correlated. As such, the number of scenarios can be significantly decreased in real-life problems. We note that existence of correlation is a special case to the problem that we discuss. Nevertheless, for large size problems, we can approximate the optimal contract amount simply by the total expected stage 1 base stock levels, \( \Omega \). To achieve this, first, we need to determine the expected bookings for the cruise liners. Namely, let \( B^l \) denote the expected number of bookings for cruise liner \( l \). It is straightforward to calculate \( B^l \)

\[
B^l = \sum_{j=1}^n q_j B^j
\]

Then we compute \( K_0^l(B^l) \) for each cruise liner \( l \) and subsequently the expected total stage 1 base stocks, \( \tilde{\Omega} \) by summing up all \( K_0^l(B^l) \) values. Ultimately, \( y_0 = \tilde{\Omega} \). Although this approximation facilitates the computations significantly, it should be underlined that the quality of it depends on the sensitivity of consumption distribution \( F \) to number of bookings \( B \) and variation across \( B \) values.

6. Summary and conclusions

The supply of food and beverage items in cruise liner operations constitutes an essential part of the services provided for the onboard vacationers. The uncertainty around effective demand and the fact that many F&B items are perishable render advance contracting for the supply of those items fairly difficult for the logistics planners. Although costly, expedited replenishments from spot markets are oftentimes necessary for sustaining required service quality. In this study, we develop a model that captures the trade-off between advance contracting and expedited replenishments of F&B items in such environments. Our model optimizes replenishment decisions under a multi-stage setting including advance contracting and expedited purchases on-departure and at intermediate stops. To determine the optimal decisions at all stages, we employ the stochastic dynamic programming paradigm that utilizes a backward induction approach. The results of the analysis suggest that while optimal stock market replenishments follow base stock policies, optimal contracting amount depends on market scenarios that lead to a piecewise cost function at the initial decision making stage.

We consider two sources of uncertainty: number of bookings at the time of contracting and demand for the F&B item over the course of the vacation. To capture demand for the F&B item during the voyage of the cruise liner, we employ a continuous, probabilistic daily consumption function whose structure depends on the number of guests onboard. On the other hand, uncertainty at initial stage for the number of guests is captured through discrete scenarios. While we base our study initially on a two-scenario model, we also discuss more generalized cases such as multi-scenario/multi-ship settings. Even though the main focus of our analysis in this paper is on cruise industry, the proposed model can be applied to various service sectors that provide non-commercial food services (e.g. in military, schools, etc.) with multiple replenishment instants with little modification if not none.

The theoretical results presented in this paper can serve as a basis in extending the research to various aspects of logistics problems on food and beverage items. It will be interesting to investigate the impact of the level of perishability on the logistical operations in the cruise line industry. Inventory and replenishment policies across multiple trips can be aggregated for items that will stay fresh for longer periods. For non-perishable items, optimization of warehousing operations must be incorporated into management of onboard inventory. Finally, another future extension worth studying is consideration of substitutability among various F&B items for optimal onboard inventory and replenishment policy analyses.

Appendix A

Proof for Lemma 2. From (11), we can derive the second derivative as follows:

\[
\frac{\partial^2 G_1}{\partial y_1} = (p - c_2)(f^{\tau_1}(y_0 + y_1 | B) - f^{\tau_1}(y_0 + y_1 - K_2 | B)) + (p + h) \left( f^{\tau_1}(K_2 | B)f^{\tau_1}(y_0 + y_1 - K_2 | B) + \int_{y_0 + y_1 - K_2} f^{\tau_1}(y_0 + y_1 - x | B)f^{\tau_1}(x | B)dx \right).
\]
Using (5), we can re-write the foregoing function
\[
\frac{\partial^2 G_1}{\partial y_1^2} = (p - c_2) f''_1(y_0 + y_1 \mid B) + (p + h) \\
\times \int_0^{m+y_1-h/2} f''_2(y_0 + y_1 - x \mid B) f''_1(x \mid B) \, dx.
\]

Clearly, the right-hand side of the above equation is strictly greater than zero implying that the stage 1 cost function is convex \(y_1\). □

**Proof for Lemma 3.** First observe that \(F'(x/B_H) > F'(x/B_L)\) for all \(x \in (0, \infty)\) from the assumption of stochastic ordering.

Hence, the derivative at the right-hand side of (2) returns a negative value at \(K_2(B_L)\) for number of bookings at \(B_H\). Since \(G_2\) is convex, the first-order optimality condition is satisfied at a higher value for \(B_H\). Therefore, \(K_2(B_H) > K_2(B_L)\). For stage 1, consider Eq. (13). Clearly, the condition in (13) is satisfied at \(K_1(B_L)\) for \(B_L\). Let us take the derivative of the left-hand side of the equation with respect to \(B\)
\[
(p - c_2) \frac{\partial F''_1(K_1 \mid B)}{\partial B} - (p + h) \frac{\partial F''_2(K_2 \mid B) F''_1(K_1 - K_2)}{\partial B} \\
+ (p + h) \int_{K_1 - K_2}^{K_1} \frac{\partial F''_2(K_1 - x \mid B)}{\partial B} F''_1(x \mid B) \, dx \\
+ (p + h) \int_0^{K_1 - K_2} \frac{\partial F''_2(K_1 - x \mid B)}{\partial B} F''_1(x \mid B) \, dx
\]

Notice that first and third terms in the foregoing equation are negative since the c.d.f. decreases in \(B\) from the assumption of stochastic dominance, second term is negative as \(K_2\) increases in \(B\). Using integration by parts we can re-write the last term as follows:
\[
\int_0^{K_1 - K_2} \frac{\partial F''_2(K_1 - x \mid B)}{\partial B} F''_1(x \mid B) \, dx \\
= \frac{\partial F''_2(K_2 \mid B)}{\partial B} F''_1(K_1 - K_2 \mid B) \\
+ \int_0^{K_1 - K_2} \frac{\partial F''_2(K_1 - x \mid B)}{\partial B} f''_1(x \mid B) \, dx.
\]

From (4), the first term at the right-hand side in the above equation is zero and the second term is negative. Consequently, the left-hand side in (13) decreases in \(B\). Since the stage 1 cost function is convex, a higher value of \(K_1\), will satisfy the optimality conditions for an increased value of \(B\). Hence, \(K_1(B_H) > K_1(B_L)\). □

**References**


