Contract optimization with front-end fare discounts for airline corporate deals

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Abstract

This paper develops a non-linear programming model to design optimal corporate contracts for airlines stipulating front-end discounts for all nets, which are defined by combination of routes, cabin types, and fare classes. The airline’s profit is modeled using a multinomial logit function that captures the client’s choice behavior in a competitive market. Alternative formulations are employed to investigate the impact of price elasticity, demand, and competition on optimal discounting policies. A case study involving a major carrier is presented to demonstrate the model. The results indicate that airlines can increase revenues significantly by optimizing corporate contracts using the suggested model.

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1. Introduction

Large corporations with high levels of traveling needs often contract with one or more airlines to receive travel discounts and other incentives. On the other hand, airlines are constantly looking to increase their corporate travel as it accounts for the majority of their operating earnings. While corporate travel accounts for approximately 55% of total air travel passengers (Granados et al., 2005), according to http://theairline-news.com they are responsible for more than two-thirds of airline revenues. In the past, corporate deals were a privilege only enjoyed by the major airlines; today however, the entry of low-cost carriers into more business markets and their increased efforts to penetrate the corporate travel market is, more than ever, prompting companies to reconsider their supplier portfolios. Though such airlines primarily are utilized through spot-buys, a growing number of companies are considering them as secondary preferred carriers. Top executives at low-cost carriers happily point out that more corporations proactively are seeking ways to include discount...
airlines in their programs, especially as these carriers grow at several times the rate of legacy airlines and invade major carrier hubs and business markets around the US.

This trend is further exacerbated by the emergence of e-commerce that has enabled companies to have access to a larger number of low-cost smaller airlines with lower overhead and transaction costs. In this new era of increased competition and stringent customer service requirements airlines are compelled to provide competitive yet cost efficient deals to their business customers. In this environment where the Internet has been moving from simply an information medium to a medium of transactions and managing business travel (Smith et al., 2001), efficient decision support systems and tools that can help develop optimal policies are in high demand. Airlines that recognize the new challenges for the corporate travel sector must be willing to develop creative programs or offer enticements so as to at least hold onto their business. Consequently, robust decision support systems and tools are needed to fully capture the emerging business models in the corporate air travel sector.

Corporate dealing is generally a function within the sales division of an airline. Account managers are assigned either to a single major corporate account or to multiple medium size and small accounts. The account manager is in charge of setting and negotiating the corporate contract, tracking the contract performance, and managing the relationship with the client’s travel management team. A contract between the airline and a corporate customer is defined by the set of discounts/incentives that the airline offers to the customer for each net (combination of route, cabin, and fare class) that the corporation flies. The major challenges faced by account managers include the lack of data visibility needed for making intelligent decisions, the lack of decision support systems that would allow them to evaluate multiple negotiation strategies in a short period of time, and the difficulty in convincing corporate travelers to book on the preferred carrier when a competitor offers an equal or superior product at an equal or lower price.

Furthermore, today, in the airline industry, not enough emphasis is placed on the integration of the systems used to support corporate contract dealing. Many account managers still rely on computer printouts and manually generated charts to make corporate contract decisions. The industry lacks integrated decision support systems that cover the entire spectrum of corporate dealing including customer value segmentation, contract design, contract optimization, and performance management. Such integrated systems should address the following business opportunities for airlines: (i) understand the value of each corporate customer’s business to the airline, (ii) increase contract compliance to drive more predictable revenue, and (iii) improve margins through decision support at the point of contract design.

An adequate integrated decision support system would further help an airline’s sales force by optimizing complex contracts involving multiple regions, maximizing margin contributions, and monitoring current contract performance. It should use forecasts of corporate passenger demand to determine optimal discount policies that maximize expected contract performance while minimizing risk. Such a system must consider three basic modules: customer segmentation, contract optimization, and performance management (see Fig. 1). The customer segmentation module identifies customers who provide predictable and irreplaceable businesses to ensure maximum return on sales efforts considering customers’ travel policies and the competitive environment. On the other hand, the contract optimization module takes into account the historical travel for each corporation, it forecasts future demand, and then builds market response functions to estimate the tradeoff between costs and revenues. Finally, the performance management module tracks contract compliance, identifies underperforming and overperforming markets, and helps the sales force to develop and implement corrective actions.

![Fig. 1. The Airline Contract Solution contains three main modules to handle contract design and tracking.](image-url)
between price, market share, and demand. Once these functions are developed, an optimization engine would generate optimal discounts for each route–cabin–class combination in each corporate contract based on revenue management and sales guidelines. Lastly, the performance management module improves contract compliance against targets and objectives by identifying deals that are under-performing or over-performing. This module provides root cause analyses of individual and team performance within the sales organization.

In this paper we focus our analysis on the contract optimization module. The purpose of this paper is to highlight the importance of a decision support system in designing airline corporate contracts and demonstrate the economical benefits the airlines are leaving on the table by not applying optimization in this field. Our study examines optimal contract design policies that include offering efficient front-end travel discount deals to corporate customers at each net. We introduce a profit maximizing non-linear mathematical programming model that determines optimal price discounts for a corporate account over all serviced nets given predetermined corporate requirements/restrictions and airline revenue management and sales guidelines. The non-linearity of the objective function is due to market share mapping that is modeled by a multinomial logit (MNL) function. The MNL function captures successfully the corporate choice behavior for all nets taking into account the impact of the competition. The proposed model can also be used to investigate the impact of price elasticities, demand structure, costs, service quality, and airline competition on an airline’s optimal corporate discount levels. Our results reveal that the interplay between optimal discount policies at different nets is more significant in the context of corporate travel in that optimal discount decisions at all nets are sensitive to changes in demand and cost parameters in one net.

The rest of the paper is organized as follows. Next section reviews the related literature. In Section 3, we provide a description of the basics of airline corporate contract deals. Next, we introduce a market response model that captures the corporate response to the airline’s offers in Section 4. In Section 5 we propose an optimization-based model with four different instances: first we investigate the unconstrained problem and then three different instances with three distinct corporate restrictions as practiced today in industry. For each of these instances we present the details of the solution methodology and discuss extensively the obtained managerial insights. In Section 6, we present a case study involving a major international carrier. Section 7 concludes the paper with summary and potential areas of future research.

2. Literature review

Most of the research in competitive airline price optimization has focused on defining optimal prices for B2C markets (business to customer) and consistently most revenue management systems are designed in this context. “Revenue Management”, the process by which the discount fares are allocated to scheduled flights for the purpose of balancing demand and increasing revenues, has been adopted by most airlines since the deregulation of the airline industry in 1978 (Pfeifer, 1989). This B2C pricing strategy has developed in part as a response by major carriers to price competition from low-cost airlines. By offering a limited number of seats at discount fares, airlines can be competitive in price with low-cost carriers and might be able to fill otherwise empty seats (Belobaba, 1987). A comprehensive taxonomy of revenue management problems is presented by Weatherford and Bodily (1992). For a recent overview of pricing models for revenue management we refer the reader to Barnhart et al. (2003) and Bitran and Caldentey (2003). Several papers that study competitive pricing in the airline industry incorporate an explicit modeling of customer choice behavior as a function of price and/or service quality. Andersson (1998) discusses the application of logit choice models to estimate buy-up (buying a higher fare after lower fares are no more available) behaviors of passengers at one of Scandinavian Airline Systems’ hubs. Algers and Besser (2001) study customer choice probabilities for flights and fare classes using revealed and stated passenger preference data. A related study on this subject is provided by Talluri and van Ryzin (2004) where the authors estimate multinomial logit (MNL) choice probabilities for fare classes in a single-leg flight via the maximum likelihood method. The authors investigate optimal policies for offering fare products at each point in time by solving a control problem. Suzuki et al. (2004) also employ MNL probabilities to model passenger choice behaviors to evaluate the impact of airfares in a region on “airport leakages”. They investigate optimal airfare policies for an airline at a certain airport via simulation. Other related papers that study airline pricing strategies include Toh et al. (1986), Bennett and Boyer (1990), Dresner and Tretheway (1992), Botimer (1996), Zhang and Cooper (2004) and Netessine and Shumsky (2005).
All the aforementioned papers consider a general B2C framework. Although some of the results from these research efforts are still valid for B2B (business to business) settings, in most part, they fall short of incorporating unique characteristics and requirements of corporate traveling in their models. Surprisingly, there are only a few papers in the literature that study corporate deals and services in the airline industry even though business travelers form the largest part of the most profitable sectors of the air travel market. To our knowledge all of the papers in this venue analyze the business travel problem from the perspective of the business traveler (buyer) rather than the airline company (supplier). In a recent paper, Suzuki and Walter (2001) study the effective use of frequent flyer programs offered by almost all airliners that can enable significant air travel cost savings for a firm. The authors propose an integer programming model that maximizes the cost savings for the firm under a deterministic setting where the firm’s all future traveling plans are known a priori. Later, Suzuki (2002) extends this model to the case where the business travelers’ future trip plans are stochastic. He proposes a heuristic approach that can be used to determine which airline to choose for each trip and when to redeem the earned miles. The airline selection problem is also addressed by Degraeve et al. (2004) from the perspective of the firm. Instead of frequent flier programs, their work considers regimes such as front-end discounts, absolute-volume discounts, and market share discounts as the base of the firm’s airline selection problem. In this approach, the authors develop a mixed integer programming model that incorporates the activity based costing hierarchy allowing for variable costs at different levels in the organization. They present a case study conducted at Alcatel Bell.

In contrast to the B2B papers discussed above, our study considers a contract optimization problem from the perspective of the airline company that offers front-end discounts for its corporate accounts across multiple legs. The proposed approach is novel in that it integrates both the model of the corporate customer and the competition into the optimization problem through revenue management and sales guidelines, corporate specific restrictions, and market response mapping. The proposed model combines the discounting decisions at all nets served by the airline in a monolithic framework.

3. Basics of corporate contracts

In the contracting process, corporations submit RFPs (request for proposal) to the airline, describing the specific routes, cabin, and classes they generally fly, and the airline in return offers incentives to try to win their business. These incentives take the form of front-end fare discounts (discounts that are offered at the booking time), back-end fare discounts (discounts that are offered at the end of the life of the contract providing the customer hit a target volume), or additional benefits (VIP lodges, limos, upgrades, etc). For small and mid-sized corporations, airlines may offer different programs that allow the corporations to earn points in proportion to travel miles in addition to the personal frequent flyer rewards given to the traveling company employees. These points can then be redeemed with round trips tickets worldwide in first class, business or coach, VIP lodges passes and memberships, and upgrades. Once a contract between the airline and the corporation is in place, it is generally adjusted and renewed on an annual basis.

In the current corporate dealing process at most major airlines, account managers create different scenarios by predicting the behavior of the corporate customer. Rules of thumb, market competition prices, and customer requests generally guide the discounts that they offer. Currently, this process does not account for seasonal traveling behaviors, specific route cost, and contribution margin for each flight segment. Once the account managers determine what they consider “the right deal”, the draft contract is sent for approval to higher echelon decision makers in the airline. Once it is approved, the sales team negotiates the final details with the corporate travel managers. When the contract is in place, the sales team is responsible for monitoring the performance of the deal; this includes measuring contract compliance against targets and objectives and identifying deals that are under-performing or over-performing. For the airline to gain and keep the corporation as a client, it should propose a deal that satisfies both its own and the corporation’s targets and objectives. From the airline’s point of view, deals are generally evaluated using the metrics that include volume, cost of deal, final revenue, and contribution margin volume measures the total number of passengers flying on nets. Cost of deal includes the discounts and commission/booking costs and it is expressed as a percentage of the contract gross revenue. Final revenue represents the total revenues that the airline receives from a corporate contract. It is computed by subtracting discount and the booking payments/commission costs from the gross
revenue (published fare times volume). Finally, contribution margin is the money earned by the airline after subtracting the route cost (i.e., final revenue minus route cost). The route cost is comprised of the incremental and dedicated costs. The incremental costs are a function of volume including dealing costs (e.g., sales force salaries), food, and a portion of fuel expenses. The dedicated route costs are the fixed costs that encompass: (i) the cost to fly the airplane; (ii) the terminal costs such as check-in, baggage handling, gate fees, etc.; (iii) the overhead costs such as corporate costs and other support staff (maintenance, janitors, admin, etc.), and (iv) the general uplift costs such as FFPS, call centers, etc.

On the other hand, from the corporation’s point of view, an attractive deal promises protection of top routes (routes with high volume), substantial savings in travel cost, and high average discounts. Basically, top routes are those with large volume, where the customer perceives the greatest value of a corporate deal. Generally, prospective clients define a minimum discount in the top routes and the airline plans around them.

Optimal design of corporate contracts can be developed through an optimization-based integrated corporate contract decision support system. The integrated framework components include three modules: (i) demand forecasting, (ii) market response model, and (iii) price optimization model.

The demand forecasting module applies seasonal forecasting algorithms (e.g. Holt Winters) to achieve a highly accurate corporation seasonal flying behavior forecast on which real-time contract performance might be predicted. Improving contract performance clearly begins with understanding future demand. The travel behavior of a corporation in each route is driven by the different businesses they have, the geographies where those businesses take place, the number of employees, number of their subsidiaries, the financial status of the corporation, etc. In few organizations an individual or a department is in charge of booking the travel for the entire company, while in others individual employees book their own travel. Travel policies in some corporations constrain the booking choices to either the lowest rate, or to any rate that falls within a range determined by the corporation’s management.

In determining optimal discounts, the airline must first understand the travel pattern of the corporation in each contracted net. With the help of historical data, and standard forecasting methods, highly accurate seasonal corporation flying behavior can be obtained. A wide variety of forecasting methods are available to airline management (see Makridakis et al., 1998). Account managers generally maintain a close business relationship with the travel department at the corporation, allowing them to get additional information and insights about critical travel behavior changes that may not be captured with historical data. In this paper, the main focus of our analysis is rather on the market response and optimization modules.

4. The market response model

This model generates corporate-specific airline market share functions that capture the airline market share response to changes in discount levels based on the state of the competition, customer preferences, and historical price data. Any market share model must capture the price elasticity of the demand, which is directly related to the possibilities of substitution for that product. A relatively large number of substitutes will imply a high price elasticity, whereas a lack of substitutes will likely force demand to become more rigid so that demand for the product may become relatively inelastic. Quite often, many carriers compete with each other on the same route, providing a case for intra-modal substitution. Although travelers also face inter-modal substitution, in corporate travel, usually intra-modal substitution is considered much more significant in affecting the demand for a particular airline.

Predictably, the market share of a given airline for a corporation is determined by the discount and quality of service that the airline offers, as well as the discount and quality of service offered by the competing airlines. A typical form of a market response curve is shown in Fig. 2. When the discount offered by the airline exceeds the discounts offered by the competition, the airline expects to get a higher market share. There is a minimum and maximum market share that the airline expects as in many corporations the employees choose the airline at the time of booking. Provided that the price complies with the travel policy of the corporation, the employee usually selects his/her favorite airline or the one with which he/she has a frequent flyer program.

To model the market share function, we employ the multinomial logit (MNL) model. The MNL model is widely used in travel demand marketing and forecasting in both literature and practice. Since the intra-modal substitution approach in general satisfies the independence from irrelevant alternatives property (Ben-Akiva
and Lerman, 1985), the MNL model is commonly used in modeling the market share among airlines as a function of price and service quality. MNL models require that each airline’s market share is non-negative and the sum of market shares of all airlines on a given net is unity. As such they satisfy the logical-consistency requirements and can efficiently capture the S-shape market share curve. The accuracy of the model depends on the available data and estimation techniques employed. The parameters of the MNL model are generally computed based on historical data. For new contracts, the historical data is replaced by data of corporations of similar size, industry, and business orientation. Several estimation approaches such as maximum likelihood, least squares, regression, and integral calculus are discussed in this context by Ben-Akiva and Lerman (1985) and Hsu and Wilcox (2000). An excellent example in the context of airline operations is provided by Suzuki et al. (2001), where the authors use the standard least squares method to estimate the MNL parameters to model an airline company’s market share. Train (2003) discusses the estimation of the choice probabilities for various discrete-choice models using simulation-based methods. Although the accuracy of the estimates is important, the main benefit of the MNL models is that they provide meaningful managerial insights regarding market-share elasticities and sales-volume elasticities (Basuroy and Nguyen, 1998).

In this study, as we focus our analysis on the impact of airfares discounts on airline corporate contract metrics, our model assumes that all other parameters related to customer service quality are exogenous. In this context, discount is defined as the percentage reduction from the published fare. Let \( d_k^i \) denote the discount offered by airline \( k \) for net \( i \). Then, by definition, \( 0 \leq d_k^i < 1 \). Now, let \( A_i \) represent the set of airlines that have a contract with the corporation for net \( i \). Given discount levels of all airlines in \( A_i \), the market share for airline \( k \) on net \( i \) is determined by the market response function, \( f_k^i \) as follows:

\[
f_k^i = \frac{e^{\beta_k^i + \gamma \ln \left( \frac{1}{1 - d_k^i} \right)}}{\sum_{j \in A_i} e^{\beta_j^i + \gamma \ln \left( \frac{1}{1 - d_j^i} \right)}}
\]

In the foregoing function, \( \beta_k^i \) is used to model all non-airfare factors such as customer service quality, passenger trip length, service frequency, safety records, and intangible non-monetary parameters. Certainly, the non-airfare factors can differ for each airline/net/corporation combination. Since the market response function includes the attributes of all airlines, any airline using this function needs historical data or expert opinion to estimate not only its own parameters but also attributes of other airlines operation on the same nets. In the absence of sufficient historical data and expert opinion, the other airlines’ market share parameters can be aggregated using the client’s total demand information and the airline’s own performance data.\(^1\)

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\(^1\) We note that in this case, the MNL model would be reduced to a binary logit model where flying with other airlines is aggregated to a single choice for each net.
Clearly, when all the airlines serving the net select the same discount level, their market share will be mainly determined by the non-airfare factors. The impact of the discount in the market response function is captured by $c$. We refer to this parameter as the \textit{price elasticity factor}, which is assumed to be uniform across all nets. This assumption is justified as we are dealing with business and not leisure travel. Higher values for $c$ imply more drastic customer reactions to price discounts as illustrated in Fig. 3. When $c \approx 1$, the market response function is convex in the discount level. The S-shape becomes more apparent as $c$ increases beyond 1.

Fig. 3 exemplifies the market share function for an airline with two competitors employing discount levels of 25\% and 30\%. Observe that below the competitors’ discount levels, the airline loses more market share as $c$ increases. On the other side, for any discount rate above the competitors’ rates, the airline’s market share increases in $c$. The situation $c \to \infty$ implies that the customer is “merely” a price-taker and thus, by outbidding the highest competition discount the airline can get all the demand. In contrast to B2C settings, both $c \approx 1$ and $c \to \infty$ cases are unlikely under B2B contracts. In general, $c$ is strictly greater than one and its value is usually higher for small to midsize businesses compared to large corporations.

As our analysis tackles the contract design problem for a single airline, for brevity, we denote the other airlines’ discount weights as

$$
\gamma_i = \sum_{j \in A / k} e^j \frac{1}{1-d_j} \gamma_i
$$

We refer to $\gamma_i$ as the \textit{collective appeal of other airlines} on net $i$. Clearly, as $\gamma_i$ increases, the airline’s market share decreases. Additionally, we drop the airline index from the function and re-write the market share function for net $i$ as follows:

$$
f_i = \frac{e^\beta_i \gamma_i \ln \left( \frac{1}{1-d_i} \right)}{e^\beta_i \gamma_i \ln \left( \frac{1}{1-d_i} \right) + \gamma_i}
$$

5. Contract discount optimization

The contract discount optimization model determines the optimal discount rate for each net. This involves analyzing hundreds of thousands of combinations to select those that are optimally suited for a given cus-
tomer’s contract at a specific point in time. The objective of the corporate contract optimization problem is to set an optimal discount level for each net based on the forecasted demand, the market response mapping, revenue management guidelines, and corporate-specific restrictions. The airline company’s goal is to maximize total profits by choosing the appropriate discounts for each net. The profit for the airline from net $i$ can be computed using the following function:

$$P_i = D_i f_i (P_i (1 - d_i) - (CC_i + RC_i))$$

(4)

In the profit function, $D_i$ and $f_i$ denote the forecasted demand of the corporation and the market share of the airline on net $i$, respectively. $P_i$, $CC_i$, and $RC_i$ are the published airfare, the route cost per passenger, and the commission and booking cost per ticket on net $i$ respectively. Let $N$ denote the set of all nets included in the contract. Then the airline aims to maximize $\sum_{i \in N} P_i$.

Note that in this model the total travel demand on any net by the corporate is assumed to be constant and not a function of the offered discounts. This assumption is justified by the fact that in corporate travel the demand is created by the corporation travel needs (e.g. the projects and sales that they are actively working on different regions) not price. As opposed to leisure travel where price motivate more customer to fly, in the business world for a typical medium to large size corporation the total travel does not present high variations based on price in that the corporation will not usually send additional employees for sales and projects just because price discounts are offered. The price mainly affects the choice of carrier.

5.1. The unconstrained case

We first analyze the maximization problem for each net separately ignoring the constraints imposed by revenue management guidelines and corporate restrictions. Solutions to the unconstrained problems can be used as a benchmark against the general case where discounting decisions must be made jointly due to constraints. In this case, the problem can be easily decomposed into nets since the profit functions for all nets are independent from each other. To determine the optimal discounting policy we first establish that the profit function for each net has a unique maximizer by showing that the profit function is unimodal in $d_i \in [0, 1)$.

**Proposition 1.** Given $\gamma > 1$, the profit function $\Pi_i$ is unimodal for $d_i \in [0, 1)$. Furthermore,

1. $\Pi_i$ monotonically decreases in $[0, 1)$ and thus, the optimal discount is zero (i.e., $d_i^* = 0$) if and only if

$$[P_i (\gamma - 1) - \gamma (RC_i + CC_i)] T_i - P_i e^{\beta_i} < 0$$

(5)

2. Otherwise, the unique maximizer, $d_i^*$, satisfies the following first order optimality condition:

$$\frac{f_i'}{f_i} [P_i (1 - d_i^*) - (RC_i + CC_i)] = P_i.$$  

(6)

The most important contribution of this result is that the optimal discount level can be determined using the first order optimality conditions. Although we do not have a closed form solution for the optimal amount, this can be easily found through a simple line search using (6). In addition, Proposition 1 provides us with other useful insights and observations regarding optimal discounting policies; these are summarized below:

- If the price elasticity is not sufficiently large (e.g. $\gamma \leq 1$) then it is never preferable for the airline to offer any discount to the corporation.
- The airline company should not offer any discount if the marginal profit on a net is negative or too small and/or its non-airfare related services and quality ($\beta_i$) are sufficiently high.
- Under an unconstrained setting, the optimal (front-end) discount is independent of the total demand for the net. It rather depends on the market share, which is modeled in terms of percentages, published airfare, and costs.
- The optimal discount rate on net $i$ increases in $P_i$ and $T_i$, and decreases in $CC_i$, $RC_i$, and $\beta_i$.

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2 All proofs are provided in Appendix A.
• The impact of price elasticity on the optimal discount rate is ambiguous for $c > 1$. In general, the optimal discount rate increases in $c$ if the collective appeal of other airlines is high. It may decrease when the opposite is true.

The first two of the above observations follow directly from (5). If the price elasticity is low the discounts will play a small role in the airline’s market share. Specifically, when the elasticity is less than or equal to one, the marginal increase in market share cannot achieve to offset the marginal loss on revenues. Subsequently, the discounts cannot be used to improve the airline’s earnings. Even for relatively higher values of $c$, discounts may not be preferable due to small revenue margins. If the airline can persuade travelers mainly through its non-airfare related efforts, discounts may also become unnecessary.

The last three of the observations are the direct byproduct of the first order optimality condition provided by (6). First, observe that since total demand for a certain net does not change as a function of the discount rates, the airline’s decisions are mainly determined by the market share. Second, an intuitive observation is that higher profit margins increase the airline’s market power and hence allow for higher discount rates. On the other hand, higher $T_i$ can also motivate higher discount rates. In this case the airline will be compelled to increase its discount rate to protect its share in the market. Naturally, there will be a negative correlation between the airline’s non-airfare related efforts and the discount rates it is willing to offer. Third, we observe from (6) the impact of the price elasticity on the optimal discount rate is context specific. If the competition offers relatively higher discount rates and/or better non-airfare related services, the market share of the airline will decline as the price elasticity increases. This pattern is a straightforward consequence of the multinomial logit function. To offset the market share loss, the airline needs to increase its discount offer under higher values of $c$.

5.2. Customer service constraints and revenue management guidelines

Although the unconstrained discount contract model provides valuable insights for optimal airline discounting strategies, in the real-world discount decisions have constraints imposed by revenue management guidelines and/or corporate restrictions. Revenue management guidelines are basically the route level discount restrictions. Contract guidelines bound the minimum contribution margin and maximum cost of dealing of the entire contract based on the customer segment. Corporate specific restrictions may include discount limits for the top routes, maximum deviation from current levels of discount, minimum average discount, minimum weighted average, and minimum total corporate savings. These restrictions are modeled through the so-called participation constraint(s) in the optimization problem. Any viable contract must satisfy these constraints.

5.2.1. Total-demand-averaged (TDA) discount constraint

Below, a mathematical formulation for the constrained optimization problem is provided:

$$ z = \max_d \sum_i \Pi_i, $$

s.t.  

$$ d^l_i \leq d_i \leq d^u_i \quad \forall i \in N $$

$$ \sum_{i \in N} D_id_i \geq \hat{d} $$

The objective function (7) maximizes the sum of contributions for all nets in the contract. Constraints (8) bound the discounts for each net based on revenue management guidelines, contract guidelines, and/or corporate guidelines. Eq. (9) ties all nets in the optimization model by restricting the weighted average of all new discounts to be greater than a certain threshold denoted by $\hat{d}$. Basically, the right-hand side of the inequality gives the minimum average discount per corporate flight acceptable by the customer. The threshold value can be determined by the weighted average of the discounts in the current contract with the corporation or as a result of negotiations with the corporation if the contract is new.

Clearly, the foregoing problem is a non-linear programming (NLP) model with a non-linear objective function and linear constraints. Without constraint (9), the model can be decomposed to nets easily and solved...
using (6) and bounds given in (8). If this solution satisfies constraint (9) then most of the conclusions for the unconstrained case stay valid. Otherwise, it is straightforward to conclude that constraint (9) must be binding at optimality. In this case, the optimal solution for small to midsize problems can be obtained by commercially available NLP solvers. We note that the optimization model is a separable non-linear program and as such solutions to large size problems can be approximated using Separable Programming. In a separable non-linear program, the objective function and constraints can be expressed as the sum of functions, each involving only one variable. Clearly, the objective function in (7) has this property. Basically, in Separable Programming the non-linear objective function is approximated to a piecewise linear function by partitioning each net profit function into small intervals that are determined by “grid” points (see Bazaraa et al., 1993 for details). Since the profit function for each net is unimodal, the new model can be then solved using the simplex method. Close approximations can be achieved if the number of grid points (and thus, the number of variables in the LP) is sufficiently high. Alternatively, the constraint in (9) can be moved to the objective function and then the problem could be solved via Lagrangian relaxation/decomposition. Through Lagrangian relaxation, we can obtain valuable managerial insights as discussed below. For the following proposition, recall that $d_i^*$ denotes the optimal discount amount on net $i$ under the unconstrained case and let $d_i^o$ represent the optimal discount amount for net $i$ in the constrained model.

**Proposition 2.** Following are true for the optimization model given in (7)

(i) For any net $i$, if $d_i^* < d_i^o$ then $d_i^o > d_i^*$. Otherwise, $d_i^o = d_i^*$.

(ii) At any optimal solution, for any given net $i$, if $d_i^* < \hat{d}$ then optimal discount amounts on all nets are non-decreasing in total demand on net $i$ ($D_i$). However, they are non-increasing in $D_i$ if $d_i^* > \hat{d}$.

(iii) For any given net $i$, where $P_i(1 - d_i^o) \geq CC_i + RC_i$, if $d_i^o > \hat{d}$ then the total optimal airline profits are non-decreasing in $D_i$. On the other hand, increased demand on net $i$ may degrade airline’s profits if $d_i^o < \hat{d}$.

The first observation in Proposition 2 states that the optimal discount rates in the constrained model will be in general above the optimal discount rates for the unconstrained case. In contrast to the unconstrained case, we observe that optimal discount rates can be influenced by demand on nets in the constrained model. Clearly, at optimality the airline will offer discount rates below the corporate threshold on some nets and balance the average by offering higher rates on other nets. We call nets with optimal rates below the threshold as the low discount rate (LDR) nets and above the threshold as the high discount rate (HDR) nets. If demand on a LDR net increases the current average discount rate will decline. As such, the airline will need to increase its discount rates on other nets so as to bring the average back to the required level. On the other hand, if the demand increase takes place on a HDR net, the average discount rate will increase giving room for discount reductions on other nets. Typically, a net becomes a LDR net if the profit margin of the airline on this net is relatively small and/or the non-airfare related weight of the airline is sufficiently high. Both of these factors create incentives to lower the discount rates. High discount rates can be afforded when the profit margin is high or the non-airfare related weight is low.

To further exemplify these results consider an airline offering a discount contract to a potential corporate client for three nets. It is given that the collective appeal of other airlines ($\gamma_j$) on nets 1, 2, and 3 are 12, 9, and 12, respectively. On Net 1 total demand is 30 units, the published fare is $100, and unit overall cost is $50. These values are 25, $100, and $80 for Net 2 and 20, $100, and $20 for Net 3, respectively. For the example, the price elasticity factor is assumed to be 3 and the threshold discount level 30%. Moreover, we let $\beta_1 = 0.7$, $\beta_2 = 1$, and $\beta_3 = 0.5$. For brevity we do not consider any lower and upper bounds on discount rates. We note that in the given example Net 2 will likely be a LDR net since the profit margin on this net is relatively small and the non-airfare weight is relatively high. On the other hand, Net 3 is more profitable yet the airline is not as strong on its non-airfare related quality on this net. Therefore, it is a HDR net. Fig. 4a depicts the impact of changes in total demand at Net 2 on optimal discount rates. Observe that optimal discount rate for Net 2 is always below the threshold and thus, from Proposition 2, optimal discount rates on all nets must increase in demand on Net 2. This situation is illustrated in the figure. On the other hand, Net 3 discount rates are always above the threshold and thus, any increase on total demand on this net will lead to lower optimal discount rates at all nets as illustrated in Fig. 4b.
Another result contrary to the unconstrained case is that total demand on a net may influence the airline profits on other nets. We note that this occurs only when constraint in (9) is strictly binding at optimality (i.e., when the customer restrictions are tight). In such a case, Proposition 2 establishes that airline profits will increase with total demand on any HDR net. We know that increased demand on a HDR net will result in lower optimal discount rates, which will lead to lower market share. However, increase in the total HDR net demand and savings from lower discount rates will more than compensate the losses in market share. This result cannot be generalized to LDR net demands in that an increase in total demand on a LDR net does not necessarily lead to higher profits as illustrated in Fig. 5. In this example, when total demand on Net 2 is sufficiently small (i.e., <20), profit margins on this net are still high enough and thus the airline’s payoff increases in the total demand. The loss in profit margins due to higher discount rates is compensated by increased sales. However, as the total demand continues to grow the profit margin on Net 2 further diminishes.\(^3\)

5.2.2. Effective-demand-averaged (EDA) discount constraint

The constraint given in (9) stipulates a minimum average discount per corporate booking. However, the corporate client may not be interested to work with such threshold as often times its employees will not prefer this airline for all their business travels (i.e., the market shares are strictly less than one at most nets). It may be more appealing to the customer to receive an average discount cap for each booking with the airline offering the contract. Consequently, it may prefer to impose the following constraint instead of (or in addition to) constraint (9):

\[
\sum_{i \in N} f_i D_i d_i \geq \hat{d}_i
\]

\(^3\) In fact, if demand gets too high on LDR nets, optimal airline profits can become negative. In such cases, the airline company should either re-negotiate the threshold discount level with the corporate client or remove LDR nets with excessive total demand from the contract.
In order for this constraint to work, the contracting airline and the corporate travel agent must have a consensus on the demand estimates and the airline’s market share with the corporate customer. The airline can achieve this through collaborating with the client on estimating the demand and the market shares.

We note that the relation among optimal discount rates, demand, and profits under constraint (10) is similar to Proposition 2 due to the fact that \( f_{di} \) increases in \( d_i \). That is, the discount rates will increase in demand at LDR nets and decrease at HDR nets. The airline’s profits increase in demand at HDR nets and the opposite may occur at LDR rates. Since market shares are below one, the impact of demand changes will be relatively smaller with constraint (10). In the aforementioned example, the airline’s profits start to drop in Net 2 with demand above 160 flights with constraint (10) in contrast to 20 flights with constraint (9) under the same threshold values. We also point that in the same example, when net demands are 30, 20, and 25, the optimal profits for the airline are $577.67 and $650.33 under constraints (9) and (10), respectively for the same threshold value of 30%. Clearly, for this instance, the latter constraint leads to higher profits for the airline. However, total corporate savings will be $1060.82 and $741.83, respectively implying that constraint (9) is more preferable to the corporate. It should be underlined that this result cannot be generalized to all cases as explained below.

Note that in constraint (10) the discount rates are weighed based on the effective demand rather than on total demand. At any given solution, the difference between the TDA and EDA profits decreases as market shares of the airline grow in LDR nets or diminish in HDR nets under fixed demand and discount rates. This is due to the fact that in both cases the value of the left-hand side in constraint (10) increases while constraint (9) remains the same. Consequently, we should expect higher payoff for the airline with constraint (9) when market shares at LDR (HDR) nets are relatively low (high). Lower (higher) market shares are expected when the collective appeal of others is higher (lower). For instance, in the example when the collective appeal of others is lowered from 9 to 6, the airline profits under constraints (9) and (10) will be $857.3 and $798.8, respectively, indicating that the airline is better off with constraint (9) this time. On the other hand, the expected savings for the corporate are $1161.7 and $1408.3 implying that the latter constraint will perform better for the corporate customer.

5.2.3. Minimum total (MINT) discount constraint

Another constraint that can be appended to the optimization problem is due to a threshold value on corporate savings. The corporate client could demand a minimum total savings of, say \( s \), in the discount contract. This threshold value can be determined based on historical data or the performance of the currently employed contract. In this case the new constraint is given by

\[
\sum_{i \in N} f_i D_i P_i d_i \geq s.
\]

Imposing the new constraint leads to the following observations:

**Proposition 3.** For the optimization model with constraint (11):

(i) At any optimal solution, for any given net \( i \), optimal airline profits increase with \( D_i \) and \( P_i \), and decrease with \( T_i \).

(ii) Optimal discount rates are non-increasing functions of demand and price, and non-decreasing functions of \( T_i \) at all nets.

6. Case study at British Airways

In this section we demonstrate the implementation of our model on a real-world case study involving renewal of the corporate contract that British Airways negotiated with CITI Group in 2004. Numbers have been scaled to protect confidentiality. British Airways (BA) has more than 200 account managers and sales executives who manage over 100,000 contract discounts in more than 600 corporate contracts, representing 2.7 billion USD in revenue. The CITI Group contract is negotiated based on 419 different nets encompassing
destinations in all continents. Based on the size and importance of this client to BA, it is expected that the contribution margin in a deal with this customer will be more than 60% while the cost of dealing will be less than 34% on average. Top routes in this contract include LHR-JFK, LHR-EWR, LHR-ORD, LHR-FRA, LHR-ZRH, LHR-JNB, LHR-HKG, and LHR-NRT, for which the current level of discount (based on the 2003 contract) cannot be decreased. Hence, the current discount levels for these nets were treated as lower bounds. Based on market competitive conditions, the account manager has determined that there is no reason to increase the discount for few of the routes such as LHR-SVO, LHR-DEL, LHR-FCO, LHR-MAD, LHR-BCN, LHR-DFW, and LHR-TPA. Therefore, discount rates on these nets were fixed at the current levels and thus, were not included in the model as decision variables. However, they were used in corporate participation constraints for calculations of the average discount levels. The parameters of the MNL model are estimated and calibrated based on combination of historical data, current market shares with the client, and expert opinion.

Four different scenarios were run before making a final offer to CITI Group. In Scenario 1, the discounts are optimized and only revenue management and contract guidelines are considered (upper and lower bounds on individual discount rates). Scenario 2 employs constraint (9) with a threshold value of 21% computed based on the previous year’s discount rates. In this scenario, BA guarantees a total-demand-averaged discount rate that is no less than what the current contract (2003 contract) would provide under the current demand forecast. Scenario 3 includes constraint given in (10) and the threshold value for the effective-demand-averaged is also matched with what the current contract can achieve (i.e., 23%). In Scenario 4, the constraint (11) is added to the problem so as to guarantee a minimum total savings for CITI Group. The optimal profits for Scenarios 3–5 are obtained by solving the non-linear maximization models given in Section 4 using Lagrangian decomposition. The results are compared with the 2003 contract’s performance and are summarized in Fig. 6.

The first column in Fig. 6 presents relevant statistics of the contract assuming that the 2003 contract discounts are not modified for 2004. The outcome of Scenario 1 results in increasing the discounts for 129 nets and decreasing them in 184 nets with a weighted average decrease of discounts by 12% compared to the previous year’s contract. This contract provides a 17% increase (£1.75 million) in profits for BA compared to the previous year. However, selling this contract to the customer would be very difficult for two reasons. First, their top routes have been modified and second, the discount per air travel ticket has decreased from 23% to 11%.

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4 In this case study, each net is indicated by a pair of airport codes. We refer the reader to http://www.world-airport-codes.com for explanation of all airport codes worldwide. We also note that most of British Airways flights subject to the deal are from LHR (London Heathrow Airport).
The outcome of Scenario 2 results in increased discounts at 269 nets and decreased rates at 62 nets. Overall, for 79% of all nets the discount rates were altered. This contract is expected to generate a 3% increase in demand and £11.7 million in profits, a 13% growth from the previous year. The outcome of Scenario 3 results in an expected contribution margin increase of 13.5% (i.e., £1.4 million). Discount rates are to be increased at 232 nets and decreased at 81 nets compared to last year’s contract. In this case at 75% of all the nets a change in discount rates has occurred and the TDA discount rate is decreased to 20%. In the last scenario, a new contract that guarantees a total discount of at least £7.5 million is designed. In this case, although the market share is increased by more than 10%, the considerable increase in discount rates (30% TDA rate and 34% EDA rate) limit the growth in optimal profits at £209 K, which is well below what the other contracts can achieve.

The results of this analysis were validated with the account manager of the CITI Group account and the changes based on Scenario 3 were implemented for the 2004 contract renewal. Under this contract, 50% of the total profits come from 15 nets (3.5% of the total number) including: LHR-JFK(1), LHR-EWR, LHR-SVO, LHR-JNB, LHR-MAD, LHR-NRT, LHR-BOM, LHR-DXB, LHR-CDG, LHR-ARN, LHR-LIN, LHR-BOS, LHR-MIA, LHR-JFK(2), and LHR-JFK(3). While the discount rates at the first six nets are not altered, they are increased at LHR-BOM, LHR-DXB, LHR-CDG, and LHR-ARN, and decreased at the last five nets. At 24 nets (5.7% of the total), BA expects loss when selling tickets to CITI Group employees under this contract.

The impact of the EDA threshold on BA profits is further analyzed by computing the optimal profits under varying values and results are depicted in Fig. 7. It is evident from the analysis of Scenario 1 that up to 11% threshold the optimal profits will remain unchanged at £12.084 million, a 17% increase in profits with respect to the previous contract. From this point on, the optimal profits decrease in EDA threshold in concave fashion. We note that up to the 43% level the airline can design a contract through an optimization that generates higher profits than the current contract with a 23% EDA discount rate.

7. Conclusions and future research

In this paper, we propose a non-linear mathematical model to determine the optimal design of corporate contracts that offer front-end discount offers to corporate travelers. The contribution of this paper to the literature is that this is the first paper that investigates the corporate travel deals from the perspective of the airlines and presents a comprehensive view of how optimization techniques can be applied to aid the airline designing the most cost effective contracts. We model the airline’s profit function using a multinomial logit function that captures the corporate client’s choice behavior under the impact of the competition. The discount decisions for all nets are interdependent by the various corporate restrictions that are integrated into the model as constraints. The model confirms the strong correlation between optimal discount policies at different nets in corporate contracting. It is often the case that high discount rates at some legs that are crucial for the airline imply lower or no discounts at others. In general, we show that price elasticity, total travel demand, costs, service quality, and airline competition play significant roles in the airline’s optimal corporate discounting policies.
Corporate dealing decision support tools such as the one proposed in this study can provide airlines with the means needed to create and manage successful corporate airline contracts. With these tools, airlines have the ability to develop strategies that create win–win situations for both the airline and the corporate clients. A pilot study at British Airways proved that by using such a tool in optimizing corporate contracts, the airline can increase the annual contribution margin from 3% to 5%; that is over £50,000,000.

We note that in our study, the corporate contract is limited to airfare discount offers. In many situations, contracts may include non-airfare related offers such as cabin class upgrades, limousine service, hotel gift certificates, resort certificates, and merchandise rewards. With the use of the proposed MNL function one could incorporate non-airfare related decisions into the model as well. However, a more generic model should consider jointly other discount schemes such as frequent flier programs and back-end discounts in addition to front-end discounts.

We note that deals with large companies may lead to overcrowding of some flight segments and thus shortage of capacity for profitable flyers. Future study should also address such dilution caused by multiple routes within a large contract that flow through a common station (hub). To tackle the problem related to the risk of dilution a more comprehensive model that integrates multiple contracts should be developed.

Appendix A

Proof of Proposition 1. First we need to show that \( \Pi_i \) is unimodal in \( d_i \) in interval \([0,1)\). Unimodularity implies that the profit function has at most one stationary point in the given interval. Clearly, at any stationary point, the equation given in (6) must be satisfied. We prove that the profit function is unimodal and the stationary point, if exists, gives the maximum value in \([0,1)\) by showing that at any stationary point the second derivative must be strictly negative when \( c > 1 \). First observe that the first derivative of the profit function with respect to \( d_i \) is

\[
\Pi'_i = D_i f''_i (P_i(1 - d_i) - (CC_i + RC_i)) - D_i f'_i P_i,
\]

At any stationary point, the foregoing function must return zero. We note that at this point the equation in (6) holds. Next, we can write the second derivative as follows:

\[
\Pi''_i = D_i f''_i (P_i(1 - d_i) - (CC_i + RC_i)) - 2D_i f'_i P_i,
\]

From (6), at any stationary point the value of the second derivative will be

\[
\Pi''_i = D_i P_i \left( \frac{f''_i - 2f'_i}{f'_i} \right) = D_i P_i \left( \frac{(\gamma - 1)f_i}{(1 - d_i)} \right)
\]

Clearly, the foregoing function returns a strictly negative value for \( d_i \in [0,1) \) when \( \gamma > 1 \), implying that any stationary point must be a maximizer and thus, \( \Pi_i \) must be unimodal. It is straightforward to see that \( \Pi_i \) is decreasing at \( d_i = 0 \) if inequality given in (5) holds. From unimodularity, this implies that \( \Pi_i \) must be decreasing everywhere in \([0,1)\) in that case. □

Proof of Proposition 2. We use the Lagrange multiplier to prove the first part. With Lagrange multiplier, \( \lambda \), constraint (9) can be removed and the objective function of the model can be rewritten as follows:

\[
z = \max_d L = \max_d \sum_i \Pi_i - \lambda \left( d \sum_{i \in N} D_i - \sum_{i \in N} D_id_i \right)
\]

The first order optimality conditions require that at optimality

\[
f''_i [P_i(1 - d^*_i) - (RC_i + CC_i)] - f'_i P_i + \lambda = 0 \quad \text{for all } i.
\]

From Proposition 1, we know that \( \Pi_i \) is unimodal and it is straightforward to see that \( \lambda \geq 0 \) if the unconstrained solution does not satisfy constraint (9). In such case, the objective function \( z \) must be increasing at \( d^*_i \) implying that \( d^*_i \geq d^*_i \) must be true if \( d^*_i \) is below the upper bound given in constraint (8).
than or equal to the upper bound, the optimal discount cannot be less than $d_i^u$ as this would clearly degrade the objective function value.

To prove the second part, first observe that in the Lagrange model $\sum_{j \in N} D_j (\hat{d} - d_j^u) = 0$. For some net $j$ if $d_j^u < \hat{d}$ and $D_j$ is increased, then the left-hand side in this equation becomes strictly positive. To pull the left-hand side down to zero again, the discount amount for at least one net (say net $k$) must be increased. Since each net’s profit function is unimodal with a maximizer and $\lambda \geq 0$, the left-hand side in the first derivative evaluated at this new discount value will be negative. Then, to satisfy the first order optimality condition for this net $\lambda$ must be increased. However, this will propagate positive derivative values for other nets evaluated at the original optimal solution. Consequently, discount rates at other nets with $d_j^u < d_j^o < d_j^o$ need to be increased as well, so as to satisfy the first order optimality conditions. Using similar analogy, one can see that increasing demand on any net $i$ with $d_i^o > \hat{d}$ will result in decreased discount rates at any net $j$ with $d_j^o < d_j^o < d_j^o$.

For the proof of the third part we use the Envelop Theorem which implies that the derivative of the optimal Lagrange function with respect to $D_i$ is

$$\frac{\partial L}{\partial D_i} = f_i(P(1 - d_i^o) - (CC_i + RC_i)) - \lambda(\hat{d} - d_i^o)$$

Clearly, the foregoing derivative is positive for $d_i^o > \hat{d}$ suggesting that the optimal profits will increase in $D_i$ in such case. Observe from the same function that the derivative may return a negative value implying downward slopping of profits in demand for $d_i^o < \hat{d}$ if the profit margin or market share of the airline is sufficiently small on net $i$ in the current optimal solution. \( \square \)

**Proof of Proposition 3.** Under constraint (11), the Lagrange function will be

$$L_i = \sum_i \Pi_i - \lambda_i \left( s - \sum_{i \in N} f_i D_i P_i d_i \right)$$

From the Envelop Theorem, the first derivative of the optimal Lagrange function with respect to both $D_i$ and $P_i$ are positive implying that the optimal profits increase in both parameters. It is straightforward to observe the opposite for $T_i$. For the second part observe that as demand or price at any net increases at any optimal solution the value of the left-hand side grows allowing for room to reduce the discount rates. The opposite is true for $T_i$ values. \( \square \)

**References**


